Digital filter design using the hyperbolic tangent functions[1]

Ahmet T. Basokur

Ankara Uni., Fen Fak., Jeofizik Müh. B., Beşevler, 06100 Ankara, Turkey.
e-mail: basokur@science.ankara.edu.tr

(Received 15 November 1997; accepted 22 January 1998)

Abstract: *Low- and band-pass filters can be designed by a combination of hyperbolic tangent functions in the frequency domain using the scaling and shifting theorems of the Fourier Transforms. The corresponding filter function in the time domain can be derived analytically from the frequency domain expression.*

The smoothness parameters control the slopes at the cutoff regions and permit the construction of a relatively short filter while reducing the oscillations of the filter response in the time domain. Different smoothness parameters can be chosen for the low and high cutoff frequencies in the band-pass filter design. Following the proposed scheme can easily derive the other type of the filters (e.g. high-pass).

Key Words: *Digital filter design, Low-pass filter, Band-pass filter*

INTRODUCTION

Johansen and Sorensen(1979) combined two hyperbolic tangent functions in the frequency domain to construct an analytical expression, which gives the filter coefficients for the numerical evaluation of the Hankel type integrals as follows:

$$P(f) = \frac{1}{2}\tanh\left[\frac{\pi}{a}\left(f+\frac{1}{2}\right)\right] - \frac{1}{2}\tanh\left[\frac{\pi}{a}\left(f-\frac{1}{2}\right)\right], \qquad (1)$$

where f and a denote the frequency and the 'smoothness' parameter, respectively. The smoothness parameter is a small constant, which takes values less than unity. P(f) is referred to as "P-function" and it is analytical in whole space from $-\infty$ to $+\infty$ and has no discontinuity. The shape of the P-function resembles a box-car function and consequently the corresponding time domain function resembles a sinc function. The inverse Fourier transform of (1) yields (Johansen and Sorensen, 1979)

$$P(t) = \frac{a.\sin(\pi t)}{\sinh(\pi a t)}. \qquad (2)$$

P(t) is referred as 'sinsh' function. The above Fourier transform pair has been also used by Christensen(1990), Sorensen and Christensen(1994) for the same purposes, namely as a truncation function in the frequency domain and as a interpolating function in the time domain. This paper presents a potential use of the hyperbolic tangent functions for designing low- and band-pass filters.

LOW-PASS FILTER DESIGN

An expression for a low-pass filter can be developed by making use of the following Fourier transform pair given by Bracewell(1965) (page:366)

$$\frac{i}{\sinh(\pi t)} \leftrightarrow \tanh(\pi f), \qquad (3)$$

where $i = (-1)^{1/2}$. By applying the scaling property of the Fourier transform one finds

$$h(t) = \frac{i\, a\, f_L}{\sinh(2\pi f_L t)} \leftrightarrow H(f) = \frac{1}{2}\tanh\left[\frac{\pi f}{2af_L}\right], \qquad (4)$$

where f_L is a constant and it will be used as the cutoff frequency of a low-pass filter in the subsequent development. Figure 1 shows a combination of two hyperbolic tangent functions giving the desired low-pass filter expression

$$H_L(f) = A\left\{H(f+f_L) - H(f-f_L)\right\}, \qquad (5)$$

where A is the gain of the filter. The amplitudes of the input are not modified if A is chosen being equal to the sampling time. Substituting (4) into (5) yields the final expression

[1] Paper presented at the First Congress of the Balkan Geophysical Society, September 1996, Athens, Greece.

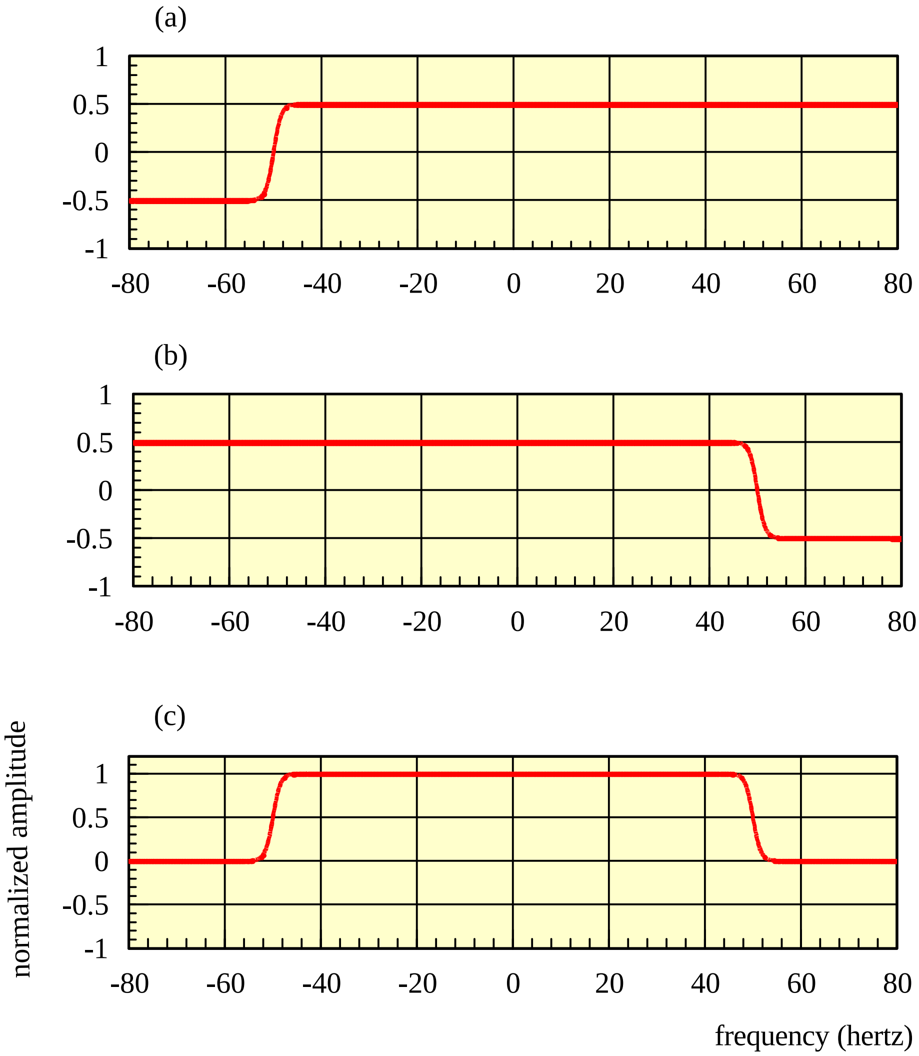

FIG. 1. Development of a low-pass hyperbolic tangent filter in the frequency domain. The combination of the functions $H(f + f_L)$ (a) and $-H(f - f_L)$ (b) derived from the equation (4) gives the desired low-pass filter $H(f)$ (c).

$$H_L(f)=\frac{A}{2}\left\{\tanh\left[\frac{\pi(f+f_L)}{2af_L}\right]-\tanh\left[\frac{\pi(f-f_L)}{2af_L}\right]\right\}, \quad (6)$$

The filter coefficients can be computed from the time domain equivalence of the equation (6). An expression for this purpose can be easily derived from (4) and (5) by using the shift theorem for Fourier transform (Bracewell, 1965)

$$h_L(t) = h(t)\, e^{-i2\pi f_L t} - h(t)\, e^{i2\pi f_L t}, \quad (7)$$

which becomes

$$h_L(t) = 2\, A\, a\, f_L \frac{\sin(2\pi f_L t)}{\sinh(2\pi a f_L t)}, \quad (8)$$

Equations (6) and (8) serve the calculation of the filter transfer function in the frequency domain and the filter response in the time domain, respectively. The smoothness parameter controls the slope of the filter transfer function around the cutoff frequency. If this parameter takes very small values, then $H_L(f)$ resembles an ideal filter represented by a box-car function (see Figure 2). Consequently, time-domain response of the filter approaches to a sinc function since $\sinh(x) \cong x$ for small arguments.

If the smoothness parameter takes relatively high values, then the slope of the filter function in the frequency domain decreases. This will significantly reduce the oscillations of the filter response function in the time-domain and permits the construction of a relatively short filter in length (Figure 3). A special care is needed in the selection of the smoothness parameter because a high value may lead unsuccessful extraction of unwanted frequency components from the input signal. However, the easy controlled degree of attenuation in the transition band is always possible.

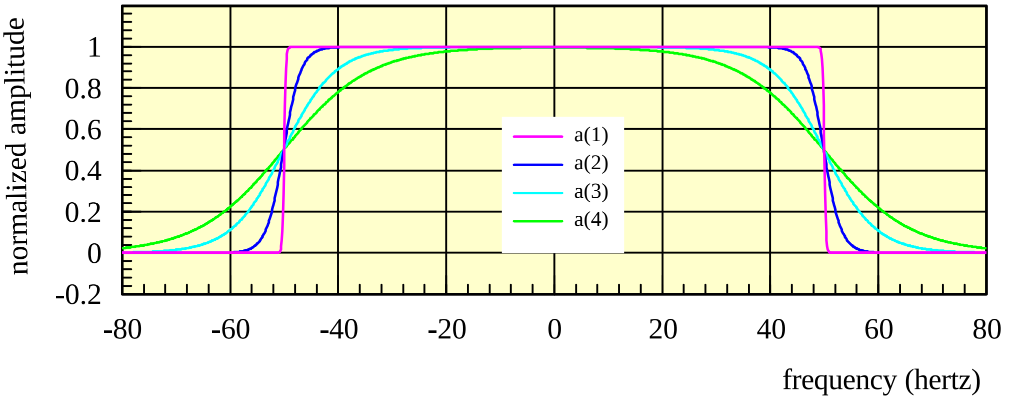

FIG. 2. Low-pass hyperbolic tangent filters obtained from the equation (6) by allowing variable smoothness parameters that control the slope of the filter transfer function. a(1) = 0.01, a(2) = 0.05, a(3) = 0.3, and a(4) = 0.5.

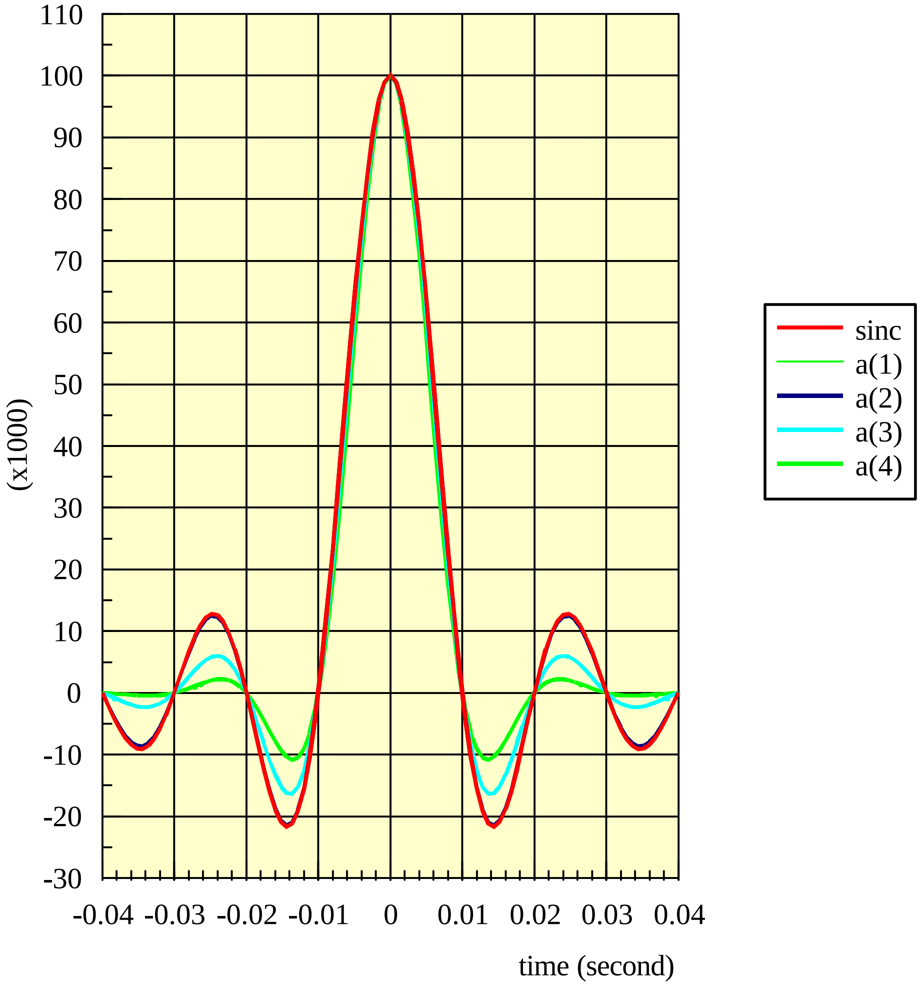

FIG. 3. The comparison of the sinc function and time domain responses of low-pass hyperbolic tangent filters given in FIG. 2. The smoothness parameters control the oscillations of the sinsh responses.

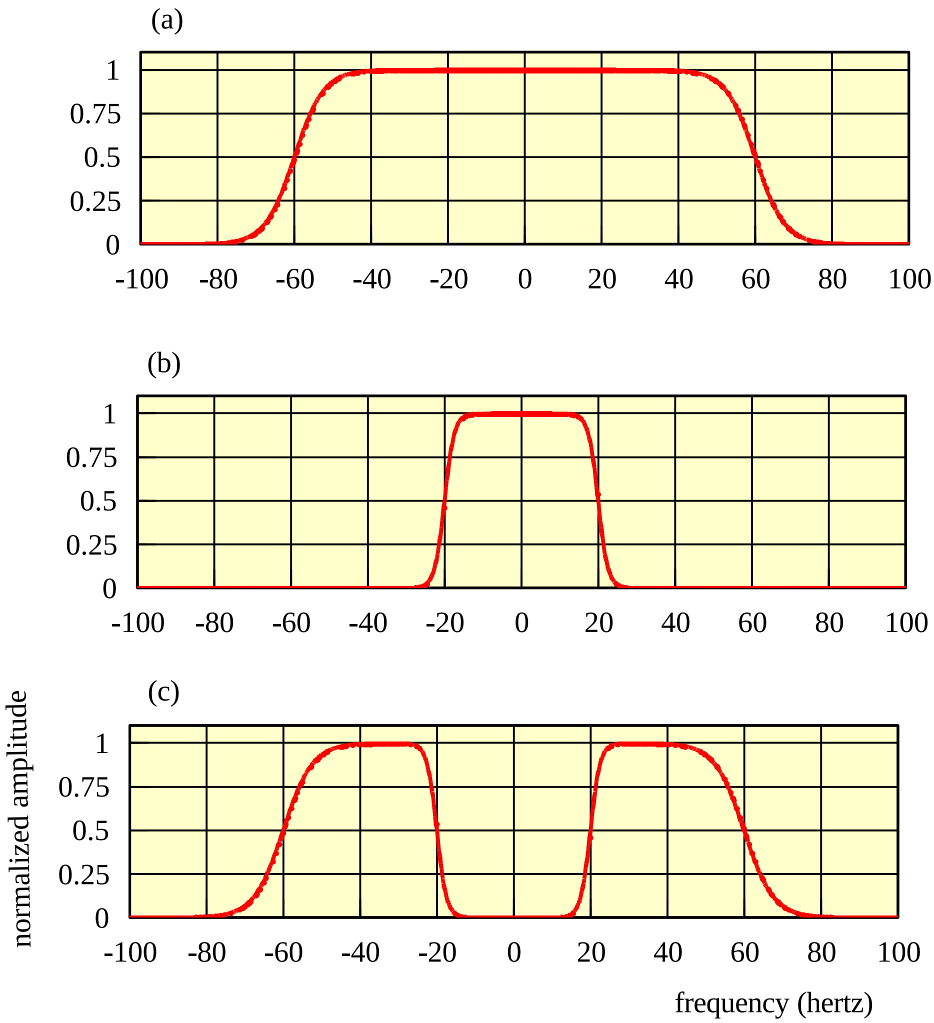

FIG. 4. Development of a band-pass hyperbolic tangent filter in the frequency domain by the summation of the low-pass filters shown in (a) and (b) having the cutoff frequency 60 and 20 Hz and the smoothness parameters 0.3 and 0.01, respectively. The resulted band-pass filter has different slopes at the low and high cutoff regions.

BAND-PASS FILTER DESIGN

A band-pass filter can be obtained by subtracting one low-pass filter from another each having different cutoff frequencies. Thus, the transfer function of a band-pass filter can be obtained by combining four tangent hyperbolic functions

$$H_B(f)=\frac{A}{2}\left\{\tanh\left[\frac{\pi(f+f_H)}{2a_2(f_H-f_L)}\right]-\tanh\left[\frac{\pi(f-f_H)}{2a_2(f_H-f_L)}\right]-\tanh\left[\frac{\pi(f+f_L)}{2a_1(f_H-f_L)}\right]+\tanh\left[\frac{\pi(f-f_L)}{2a_1(f_H-f_L)}\right]\right\}, \qquad (9)$$

where f_L and f_H are the low and high cutoff frequencies and a_1 and a_2 are smoothness parameters which determine the slopes corresponding f_L and f_H. The different numerical values of the smoothness parameters a_1 and a_2 permit the independent adjustment of the slopes in the transition bands (see Figure 4).

The inverse Fourier Transform of (9) yields the response function in the time domain

$$h_B(t)=2A(f_H-f_L)\left\{a_2\frac{\sin(2\pi f_H t)}{\sinh(2\pi a_2 t(f_H-f_L))}-a_1\frac{\sin(2\pi f_L t)}{\sinh(2\pi a_1 t(f_H-f_L))}\right\}. \qquad (10)$$

The sample values of (10) gives the desired filter coefficients (Figure 5). Other properties of the band-pass filter are the same as that of the low-pass filter.

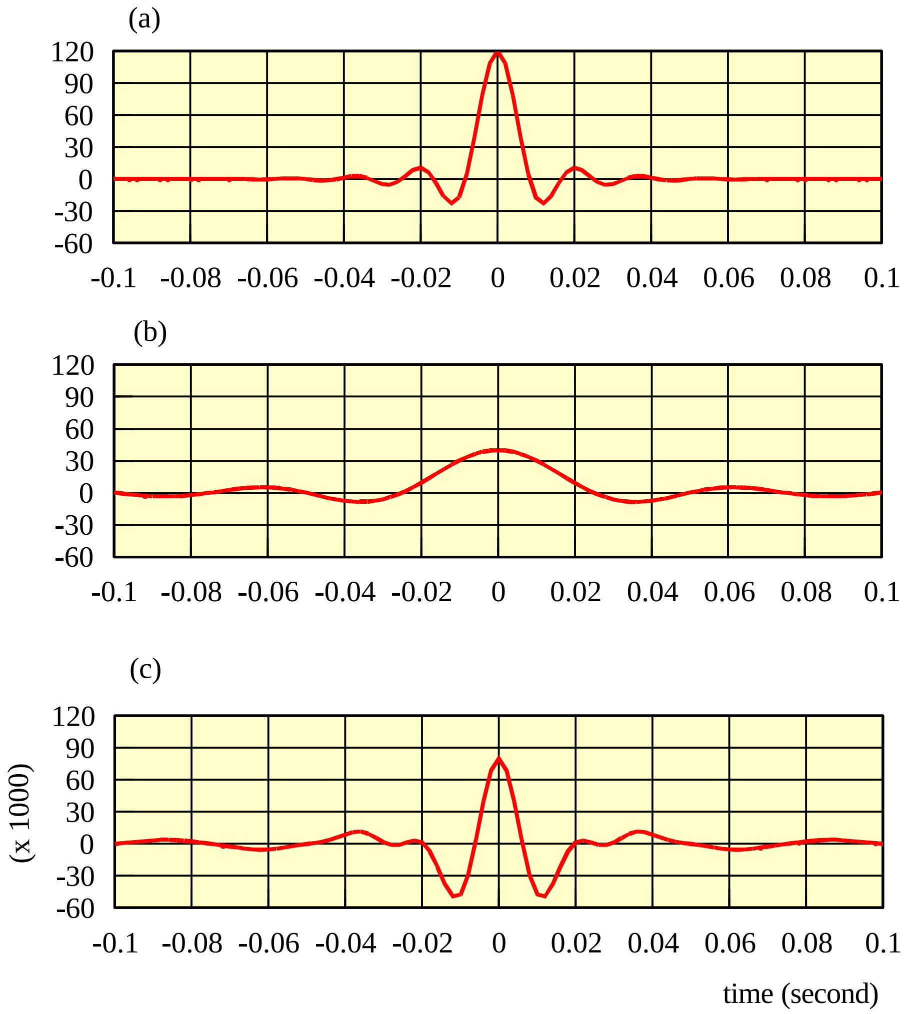

FIG. 5. Development of the time domain response of a band-pass filter. (a), (b) and (c) correspond the time domain representations of the filter transfer functions gives in FIG. 4a,b and c.

CONCLUSION

The main properties of the suggested filter can be summarised as follows:

a) The transfer function of the hyperbolic tangent filters is analytical in whole space from $-\infty$ to $+\infty$ and has no discontinuity.

b) The smoothness parameters control the slopes at the cutoff regions and different smoothness parameters can be chosen for the low and high cut-off frequencies in the band-pass filter design.

c) The corresponding filter function in the time domain can be derived analytically from the frequency domain expression and permit the construction of a relatively short filter while reducing the oscillations of the filter response in the time domain.

d) The easy control of the slope of the transfer function and the suppression of the ripples of the response function are the main advantages of the suggested filters. The decision about the value of the smoothness parameter can be made interactively depending on the given problem.

ACKNOWLEDGMENT

I am grateful to Dr. Edip Baysal for valuable discussions. This work was supported by the Scientific and Technical Research Council of Turkey (TÜBITAK) under project no. YDABÇAG-548.

REFERENCES

Bracewell, R., 1965, The Fourier transform and its applications: McGraw-Hill Book Co.

Christensen, N. B., 1990, Optimized fast Hankel transforms: Geophysical Prospecting, **38**, 545-568.

Johensen, H. K., and Sorensen, K., 1979, Fast Hankel transforms: Geophysical Prospecting, **27**, 876-901.

Sorensen, K.I., and Christensen, N. B., 1994, The fields from a finite electrical dipole - A new computational approach: Geophysics, **59**, 864-880.