

## Calculation of the azimuthal inhomogeneity ratio response of 3-D structures using an analytical forward modelling scheme

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**Abstract:** *The Azimuthal Inhomogeneity Ratio (A.I.R.) can be obtained by measuring the three possible modes of connection for resistance measurements with the square array and it reflects the presence of subterranean lateral inhomogeneities. The A.I.R. anomalies form a signal of dipolar pattern and have relatively large amplitude.*

*The expected A.I.R. pattern of two models was computed in the framework of the present study together with the respective resistance anomalies for all three possible modes. The method of "dipole sources" was applied for the computation of the response of the 3-D models. The models that simulate the targets commonly encountered in archaeological exploration were selected to test the validity of the suggested method.*

*The calculated A.I.R. patterns delineate very well the edges of the disturbing bodies. Moreover, they present an intricate pattern suitable that can be handled by the pattern recognition schemes.*

**Key Words:** *Azimuthal Inhomogeneity Ratio, Resistivity Modelling, Dipole Sources.*

### INTRODUCTION

The square arrangement of electrodes in an earth resistance measuring system has received relatively little attention in "archaeological geophysics". This is mainly due to the fact that systems involving the mobilisation of only two electrodes possess an obvious operational advantage for profiling. Thus, systems like the Schlumberger and "twin probe" (Aspinall and Lynam, 1970) electrode configurations became very popular. Nevertheless, Clark (1968) employed the square array for archaeological exploration and claimed that some operational facilities can be achieved if small spacing is used. However, all the drawbacks concerning the operational difficulties were raised by the "towed resistivity meter with automatic recording" system which was proposed by Hesse et al. (1986).

Habberjam (1975, 1979) has given an extensive analysis of the square array's merits and drawbacks, either for mapping or for probing. An advantage of the array is that two apparent resistivity values can be measured in mutually perpendicular orientations (Habberjam and Watkins, 1967). Therefore, the average of two measurements can provide a more orientationally stable measure of apparent resistivity. However, the most fasci-

nating feature seems to be the ability of the array to reveal the existence of orientational effects by the difference observed between the referred to measurements. This difference is by definition, equal to the resistance that can be measured by the third electrode arrangement. The later resistance is called  $R_\alpha$ , while the two other resistance measured at mutually perpendicular orientations are called  $R_\beta$  and  $R_\gamma$ .

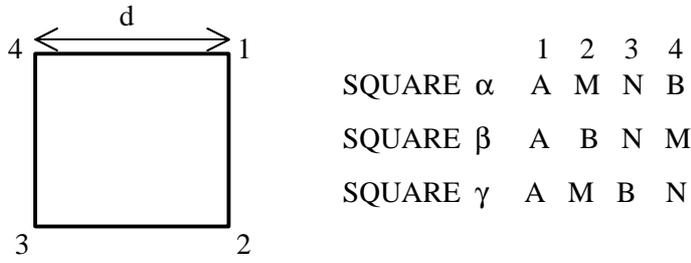
The orientational effects, that arise from the lateral inhomogeneities existing in the ground, can be expressed by the Azimuthal Inhomogeneity Ratio (A.I.R.) This is given by

$$\text{A.I.R.} = \frac{2R_\gamma(a)}{R_\alpha(a) + R_\beta(a)} \quad (1)$$

where,  $a$  is the array spacing, i.e. the length of the square. In fact, the A.I.R. is a measure of an apparent direction of resistance.

The three basic electrode arrangements which provide the  $R_\alpha$ ,  $R_\beta$  and  $R_\gamma$  resistances are named  $\alpha$ ,  $\beta$  and  $\gamma$  accordingly. These are shown in Figure 1.

Grigoriou and Tsokas (1997) studied the additional information that can be derived using the A.I.R. measurements. They employed a very stable data set in



**FIG.1.** The square configuration and the possible electrode arrangements.

which the observational errors were eliminated according to the procedure proposed by Carpenter and Habberjam (1956). Additionally, the data was corrected for the diurnal variation of moisture content. They concluded that the A.I.R. response of subsurface targets seems very promising and suitable for the application of pattern recognition techniques.

Tsokas *et al.* (1997) used a modified 2.5-D finite element method scheme in order to calculate the square array resistivity anomalies and inhomogeneity ratio of various subsurface models. They compared these results with the response of several different linear arrays and illustrated the superiority of the A.I.R. over conventional arrays for improved interpretation.

This work extends the study of the A.I.R. response to fully three-dimensional structures by making use of a particularly flexible and computationally efficient forward modelling scheme.

### THE METHOD USED TO COMPUTE THE ELECTRICAL RESPONSE OF 3-D STRUCTURES

Considering a 3-D structure buried in a homogeneous and isotropic environment of resistivity  $\rho_0$ , the apparent resistivity  $\rho_a$  measured at the surface of the earth could be expressed as

$$\rho_a = \rho_0 \frac{\mathbf{E}(\bar{x})}{\mathbf{E}_0(\bar{x})} \quad (2)$$

where  $\mathbf{E}_0(\bar{x})$  is the electric field due to homogeneous earth while  $\mathbf{E}(\bar{x})$  is the sum of the homogeneous and anomalous fields.

If we locate a current electrode at the point  $\bar{x}_c$  on the surface, the normal electric field at any point located at coordinate  $\bar{x}$  is given by the well-known formula:

$$\mathbf{E}_0(\bar{x}) = \frac{I\rho_0}{2\pi\|\bar{x} - \bar{x}_c\|^2}. \quad (3)$$

This vector can be considered as being a quantity that induces electric polarisation at any point of the 3-D structure. This concept is equivalent to magnetisation induced by the Earth's magnetic field. Suppose that the disturbing body consists of small cells of constant resistivity  $\rho(\bar{x}_0)$  where the vector  $\bar{x}_0$  defined by the

coordinates of the centre of a cell. In that case the electrical polarisation at the point  $\bar{x}_0$  can be expressed as

$$\mathbf{P}(\bar{x}_0) = q(\bar{x}_0) \left\{ \mathbf{E}_0(\bar{x}_0) + \sum_k \mathbf{E}_d(\bar{x}_k - \bar{x}_0) \right\} \quad (4)$$

where,  $\mathbf{E}_d(\bar{x}_k - \bar{x}_0)$  is the electric field of the cell  $k$  with centre coordinates  $\bar{x}_k$  at the point  $\bar{x}_0$ . The coefficient  $q$  of equation (4) is

$$q(\bar{x}_0) = \frac{3(\rho(\bar{x}) - \rho_0)}{4\pi\rho(\bar{x}_0 - 2\rho_0)}. \quad (5)$$

In case where the resistivity does not vary within the disturbing structure and has a value of  $\tilde{\rho}_1$ , the expression (5) reduces to

$$q(\bar{x}_0) = \frac{3}{4\pi} \frac{(\rho_1 - \rho_0)}{(\rho_1 - 2\rho_0)}. \quad (6)$$

Numerical experiments show that the usual iteration process to solve equation (4) diverges. The divergence is due to the numerical value of  $q$ , which is always

$$q \leq \frac{3}{4\pi}.$$

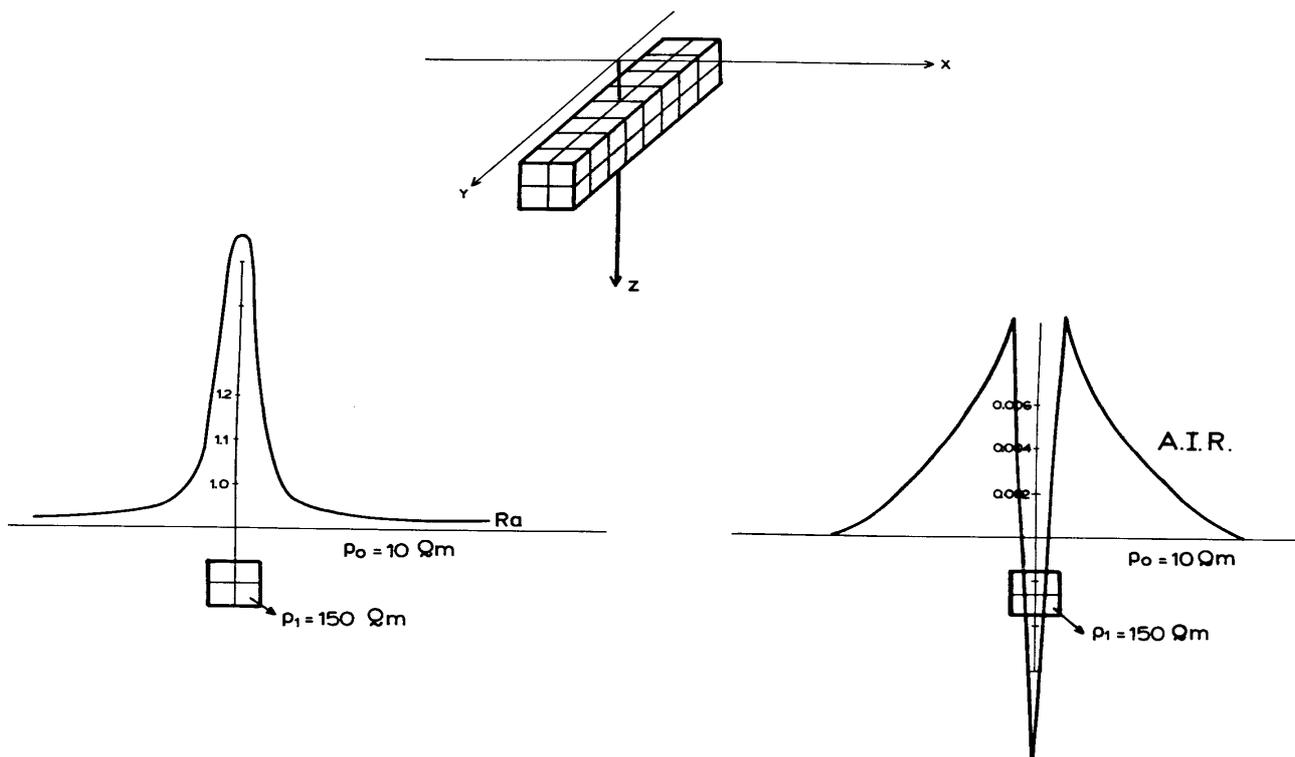
The use of the regularisation method becomes necessary to obtain convergence. Employing Zeidel's method, Ermokhin (1988) gave a recurrent formula for the regularised solution as follows:

$$\mathbf{P}_1(\tilde{x}_k) = \frac{2}{3} \mathbf{P}_{1-1}(\bar{x}_k) + \frac{1}{3} q(\bar{x}_k) \left\{ \mathbf{E}_0(\bar{x}_k) + \sum_{\substack{i=1 \\ i \neq k}} \mathbf{D}(\bar{x}_i - \bar{x}_k) \mathbf{P}_{1-1}(\bar{x}_k) dv \right\} \quad (7)$$

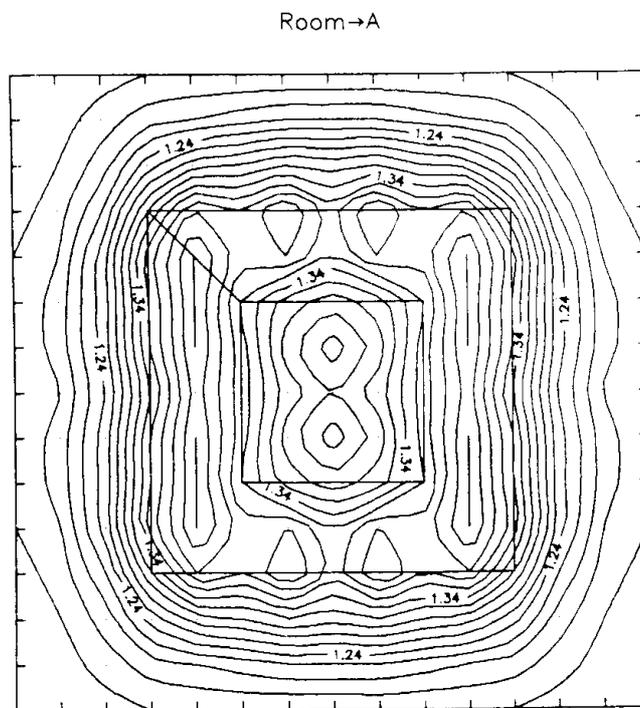
The polarisation induced by the applied field can be used as the first approach:

$$\mathbf{P}_0(\bar{x}_k) = q(\bar{x}_k) \mathbf{E}_0(\bar{x}_k). \quad (8)$$

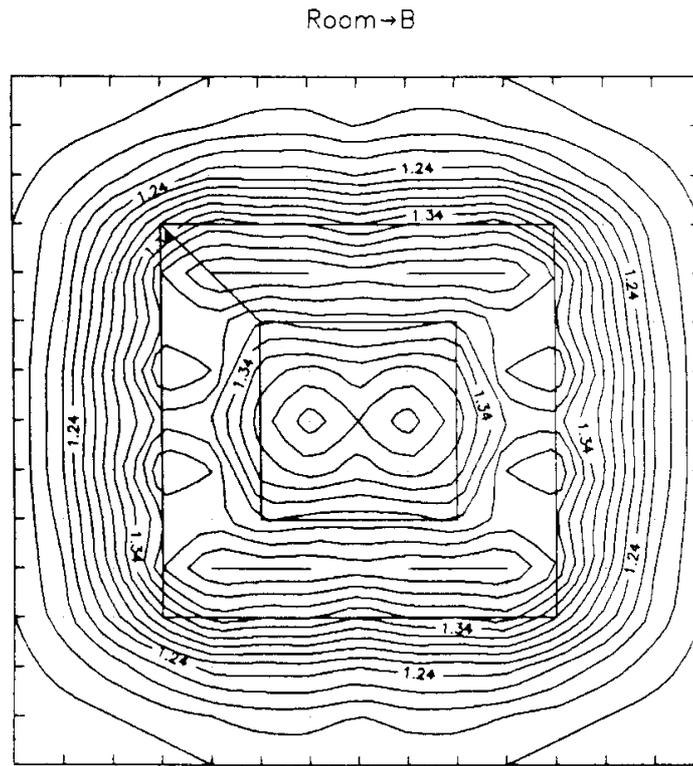
The solution of the system of equation (7) gives the polarisation vectors at the centre of each cell. These quantities allow the determination of the anomalous plus the normal electric field at point  $\bar{x}$  on the surface of the earth. In other words, the determination of the quantity  $\mathbf{E}(\bar{x})$  of equation (2) is possible. Ermokhin (1988) gives the whole procedure



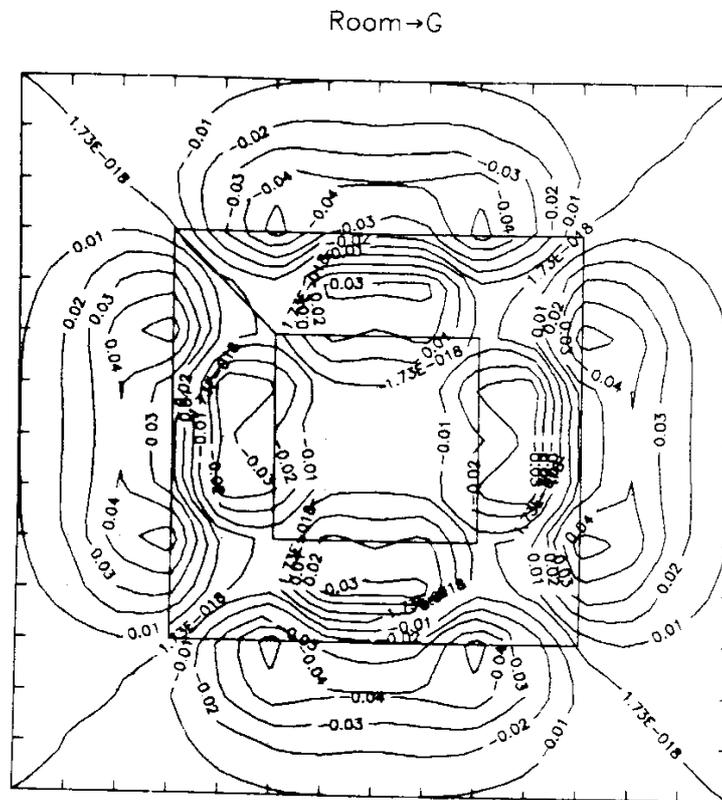
**FIG. 2.** Electrical response of a 3-D target in terms of  $R_{\alpha}$  resistance and A.I.R. The target is shown in the upper part of the Figure 1. The target has been divided into 32 cubic cells for the computation scheme. Profiling was conducted along x axis and the spacing of the square array was 1m.



**FIG. 3.** Calculated  $R_{\alpha}$  effect of a model, which simulates the ruins of a building. The structure is supposed to be buried at 0.5 m beneath the ground surface. It is consisted of parallelepipeds of  $1 \times 1 \times 5 \text{ m}^3$  and shown in plan view on the figure. The resistivity of the hosting medium is 10 ohm-m contrasting to the 100 ohm-m of the structure. The square array spacing was 1 m.



**FIG. 4.** Calculated  $R_\beta$  response of the structure shown in plan view. The parameters are the same as in Figure 3.



**FIG. 5.** Calculated  $R_\gamma$  response of the structure shown in plan view. The parameters are the same as in Figure 3.

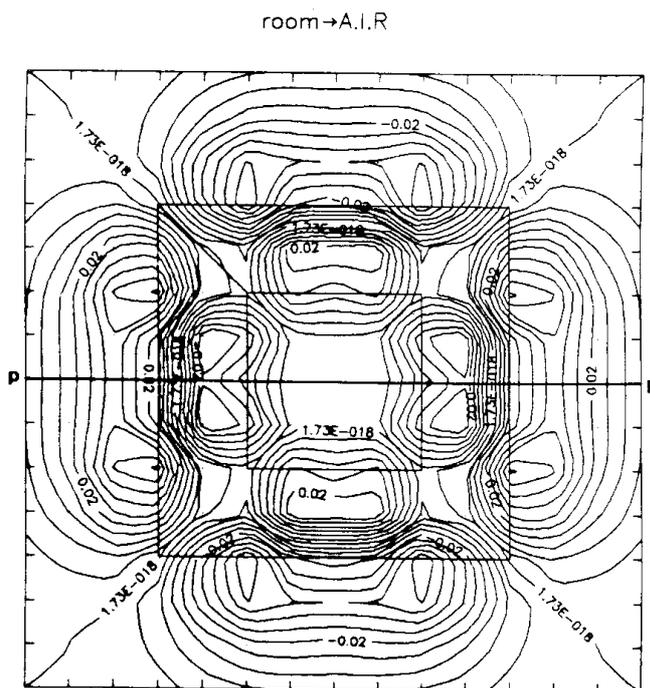


FIG. 6. The A.I.R. spatial distribution of the structure in plan view, having all other parameters the same as those used for the presentation of Figures 3, 4 and 5.

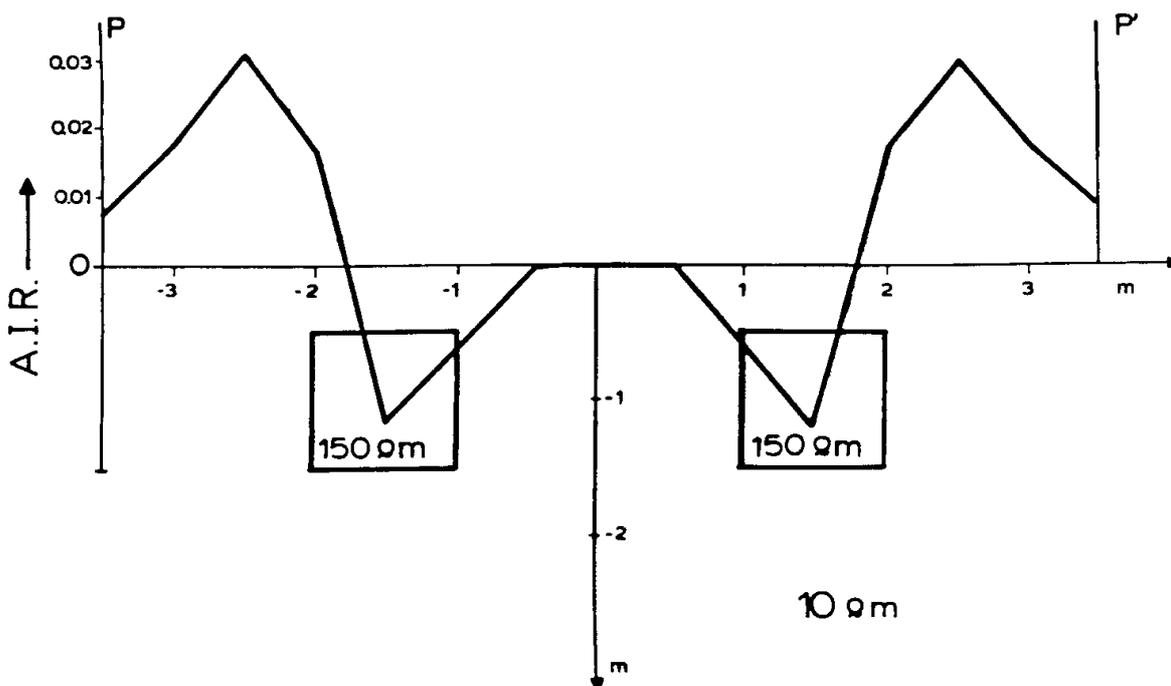


FIG. 7. The A.I.R. variation along a profile drawn from the data given in Figure 6.

presented here in details.

### COMPUTATION OF THE EFFECT OF 3-D STRUCTURES AND DISCUSSION

The most common structures in archaeological prospecting are the remains of foundation walls. Thus, the expected anomalies due to a concealed wall for each configuration mode were computed by using the analysis given in the previous section. In Figure 2, the strike of the wall coincides with the  $R_\alpha$  resistance measurement direction and the A.I.R. variation. Profiling is supposed to have been conducted along x axis and the array spacing is  $a=1\text{m}$ . The resistivity of the environment,  $\rho_o$ , was set to 10 ohm-m while the resistivity of the body which simulates the wall,  $\rho_1$ , was considered 150 ohm-m.

Figures 3, 4, 5 and 6 show the spatial distribution of the  $R_\alpha$ ,  $R_\beta$ ,  $R_\gamma$  resistances and A.I.R., respectively, which are produced by the structure shown in the plan view on the same drawings. The environment supposed to have a resistivity of 10 ohm-m while the structure a resistivity of 100 ohm-m. The burial depth of the feature is 0.5m and it is consisted of parallelepipeds  $1 \times 1 \times 5 \text{ m}^3$ . The whole feature is a realistic simulation of the buried relics of the foundation walls of a building. The orientation of the current and potential dipoles has been reflected on the anomaly patterns of Figures 3 and 4. On the other hand, the anomaly patterns of the  $R_\gamma$  resistance and A.I.R. are the same as expected.

However, the pattern, which deserves specific attention, is that of Figure 2. The A.I.R. pattern delineates exactly the edges of the disturbing body in a pronounced and indisputable manner. This fact was expected since A.I.R. is a qualitative and quantitative measurement of the apparent electrical anisotropy. By definition it ought to be zero, wherever the subsurface is isotropic with the exception of the square array being placed at  $45^\circ$  with respect to the electrical strike (Habberjam, 1975). Of course, we are dealing with structural anisotropy in the present study. In other words, we consider the anisotropy due to the presence of subsurface structures within a uniform and isotropic medium. The structures themselves are considered uniform and isotropic as well.

Quantifying the above assumptions we can write the following condition

$$R_\gamma = R_\beta$$

$$R_\gamma = 0 \quad (9)$$

which should be satisfied wherever the array samples only the host medium. Wherever the array samples a space enclosing both media, then the condition (9) would not be satisfied, and  $R_\gamma$  will have a value other than zero. In other words, the  $R_\gamma$  resistance is the indication of the physical discontinuities. Hence, it is reasonable that  $R_\gamma$  responds in a more pronounced

manner when the array crosses the edges of the structure. A signal having both positive and negative lobes appears and both peaks have large amplitude. This is easily observed in Figures 2 and 7. In the case of Figure 7, the A.I.R. variation along the profile annotated as pp' on Figure 6 has been drawn.

The dipolar nature of the A.I.R. signal is both an advantage and a drawback. In simple cases, it is an obvious advantage because we can easily conclude the existence and the exact location of a lateral inhomogeneity. In more complicated cases, the response of various inhomogeneities will appear to be superimposed one upon the other. The dipolar pattern of the A.I.R. signals would make the anomaly look more obscured than, for example, the  $R_\alpha$  anomaly pattern. Grigoriou and Tsokas (1997) cross-correlated A.I.R. profiles with calculated signals in order to better identify specific targets. The A.I.R. signal seems more suitable for cross-correlation than the respective  $R_\alpha$  signal. This is the direct consequence of its dipolar and more complicated nature.

Surveying with square array is not very productive in comparison with the conventional arrays. It is greatly time consuming if three resistances are measured instead of one. The current automated systems overcome the disadvantages that arise due to the low operational speed.

### CONCLUSIONS

The A.I.R. (Azimuthal Inhomogeneity Ratio) anomaly patterns reflect directly the edges of subsurface lateral inhomogeneities. Therefore, they show clearly the edges of the disturbing patterns, unless the array is orientated at  $45^\circ$  degrees with respect to the electrical strike.

The dipolar nature of the A.I.R. anomaly pattern makes possible the use of pattern recognition schemes.

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