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An application of the semi-Markov model for earthquake occurrences in North Anatolia, Turkey

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Abstract: *Earthquake occurrence probabilities are estimated by applying the semi-Markov Model. This model assumes that the successive earthquakes in the same structural discontinuity are not independent events, but they are influenced by the elapsed time interval between them. The North Anatolian Fault Zone between longitudes 26.00^O E and 42.00^O E is selected for this study. Seventy-one earthquakes having surface wave magnitudes $M \geq 5.5$ occurred in the region between 1902 and 1992 are considered. The occurrence probabilities were obtained by determining interval transition probabilities for region to region and magnitude to magnitude transitions of these earthquakes. The determination of joint probabilities demonstrates that earthquake occurrence can be investigated in the three dimensions of space, time and magnitude.*

Key Words: *Earthquake Occurrence, Semi-Markov Model, State Transition , Holding Time, Interval Transition Probability, North Anatolian Fault Zone.*

INTRODUCTION

The fields of seismology and earthquake engineering deal with the studies for the prevention of possible damage due to destructive earthquakes in seismic zones and the minimisation of their detrimental effects. These studies include earthquake prediction research and earthquake hazard assessment. Researchers have used various kinds of statistical models for earthquake occurrence. The most familiar model is the Poisson model for random series of events. The main assumption of this model is that earthquake occurrences are independent in space and time. The Poisson model is also frequently applied for seismicity studies (for example, Cornell, 1968; Caputo, 1974; Shah, 1975; Bath, 1978). The Extreme Value Statistics applied by several authors is also based on the Poisson model (e.g. Epstein and Lomnitz, 1966; Milne and Davenport, 1969; Schenkova and Karnik, 1970; Yegulalp and Kuo, 1974). Shlien and Toksoz (1970) considered the temporal clustering of earthquakes and used cluster analysis, while Rikitake (1975) used a Weibull distribution. Earthquake hazard studies based on the Poisson model may often yield adequate results in regional areas, but they are rarely suitable for specific and small areas (Oliveira, 1974), because earthquake occurrences in these areas are not fully independent of each other. This situation suggests a new approach by introducing the Markov model. In contrast to the Poisson model, the Markov model assumes that all events are dependent on one another in space and time. Hagiwara (1975), Kiremidjian and Anagnos (1980, 1984), and Grivas (1980) used the same model. Mogi (1969), Kelleher (1970), and Sykes (1971) suggested that large earthquakes do not occur randomly but are related to each other. Patwardhan *et al*. (1980) proposed that the magnitudes and times of earthquakes at discontinuities would have not been distributed randomly. They used the semi-Markov model in the Pacific Belt with earthquakes of magnitude equal to or greater than 7.8. Cluff *et al.* *(*1980) applied the same model to the Wasatch Fault Zone, Utah for earthquake magnitudes of $6.5 \leq M \leq 7.5$

According to the semi-Markov model, the magnitude of an earthquake depends on the magnitude of the previous earthquake and the time interval between them. This may indicate that a long period seismic quiescence may end with an earthquake of large magnitude. Application of the semi-Markov model leads to an estimation of the earthquake occurrence probabilities and may contribute to earthquake hazard studies (Altinok, 1988).

SEMI-MARKOV MODEL

The semi-Markov model is a probabilistic model useful in analysing complex dynamical systems. Its behaviour is similar to that of a pure Markov model. The semi-Markov theory involves the concepts of state and state transition. The behaviour of this process is characterised by a matrix

$$G_{ij} = \mathrm{Prob}\{s(n+1) = j | s(n) = i\} \quad ; \quad n = 0,1,2,... \qquad (1)$$

where *s*, denotes state and *n* is the number of time units. Thus, the probability of making a transition to any new state of the process depends only on the state presently occupied (Howard, 1971).

Transition Matrix

The transition probability, G_{ij} , is the probability that a semi-Markov process that has entered state *i* on its last transition will enter state *j* on its next transition. These probabilities must satisfy the conditions

$$G_{ij} \geq 0 \quad ; \; i = 1,2,...,N, \quad j = 1,2,...,N$$

and

$$\sum_{j=1}^{N} G_{ij} = 1 \quad ; \; i = 1,2,...,N \qquad (2)$$

where *N*, is the total number of states in the system.

Holding Time

Before the transition from state *i* to state *j* the process remains in state *i* for a time t_{ij} . The holding times t_{ij} are positive, integer-valued, random variables. All holding times are finite and each is at least equal to one time unit. The probability mass function T_{ij} of t_{ij} is called the holding time mass function for a transition from state *i* to *j*.

$$\mathrm{Prob}\{t_{ij} = m\} = T_{ij}(m) \; ; \; m = 1,2,...,n \qquad (3)$$

We must specify both the holding time mass functions and the transition probabilities to describe a discrete-time semi-Markov process completely.

Core Matrix

The *ij* th element of the core matrix *C(m)* is the probability of the joint event that a system that entered state *i* at time zero makes its next transition to state *j* and makes that transition after a holding time *m*.

$$C_{ij}(m) = G_{ij} T_{ij}(m) \quad ; \quad i = 1,2,...,N; \; j = 1,2,...,N; \; m = 1,2,....,n \qquad (4)$$

We denote equation (4) as congruent matrix multiplication form by

$$C(m) = G \otimes T(m) \qquad (5)$$

where the operator "$\otimes$" denotes multiplication of corresponding elements. If we sum the elements of *C(m)* across the *i* th row, we obtain the waiting time mass function $w_i(m)$ for the *i* th state

$$\sum_{j=1}^{N} C_{ij}(m) = \sum_{j=1}^{N} G_{ij} T_{ij}(m) = w_i(m) \qquad (6)$$

The cumulative probability distribution of the waiting time is obtained from

$$\leq w_i(n) = \sum_{m=1}^{n} w_i(m) \qquad (7)$$

the complementary of $\leq w_i$ is then

$$> w_i(n) = \sum_{m=n+1}^{\infty} \text{ sum of elements in } ith \text{ row of } G \otimes T(m) \qquad (8)$$

INTERVAL TRANSITION PROBABILITY MATRIX

The most important statistics of the semi-Markov process are the interval transition probabilities. The probability of a transition from state *i* to state *j* in the interval *(o,n)* requires that the process makes at least one transition during that interval. The process could have made its first transition from state *i* to some state at time *m*, where $0 \leq m \leq n$, and then by some succession of transitions could have made its way to state *j* at time *n* (Howard, 1971).

$$F(n) => W(n) + \sum_{m=0}^{n} G \otimes T(m) F(n-m) \; ; \; n = 0,1,2,... \qquad (9)$$

where $>W_n$ is the diagonal matrix with its *i* th element equal to $>w_i(n)$. The interval transition probability, *F(n)*, is obtained by a recursive process.

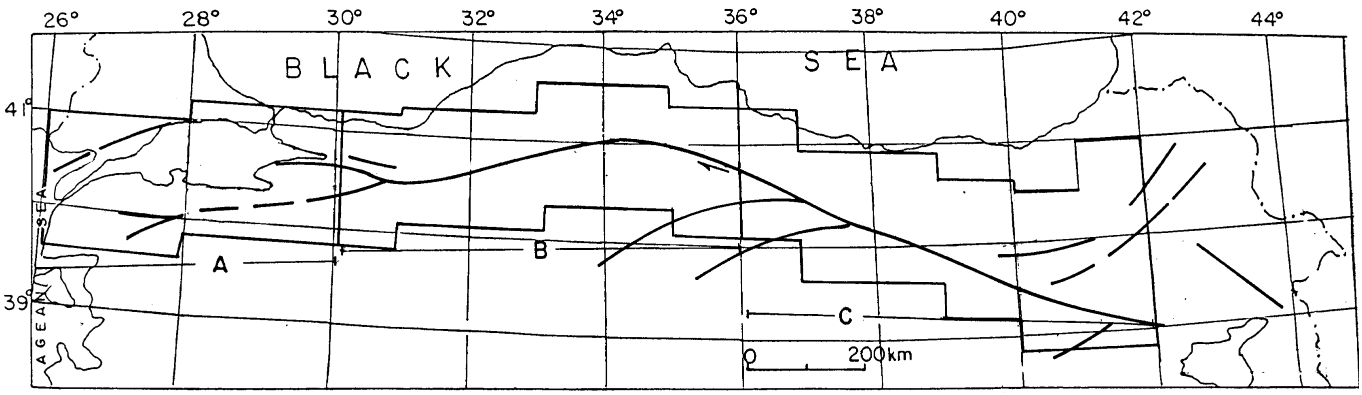

FIG. 1. The area of investigation (North Anatolian Fault Zone).

For $m=0$, $T(m)=0$ therefore $F(n)$ is obtained for the interval $1 \le m \le n$. For $n=0$, $F(n)$ is equal to the Kronecker Delta. In earthquake phenomena, it is possible to make an approach with these probabilities to earthquake hazard studies.

APPLICATION OF THE MODEL

The North Anatolian Fault and its postulated western and eastern continuations were selected as an area of investigation. This area is bounded by longitudes 26.00^{o} E and 42.00^{o} E (Fig. 1). The data of seventy-one earthquakes, which occurred between 1902-1992, were used. The earthquakes of this area are generally shallow and destructive if $M>5.0$. For this reason, the earthquakes with $M \ge 5.5$ were selected. Earthquake data were obtained from the following sources: Ayhan *et al.* (1988), Ambrasseys and Jackson (1981), Dewey (1976), Earthquake Data Set of Kandilli (1996), Eyidogan *et al*. (1991) and Bulletins of PDE, ISC.

Region to Region Transitions

From a consideration of the epicentre distributions, the continuation of fault zones and the mode and size of displacement, three regions, A, B, C were selected as states (Fig. 1). The transition probability matrix was obtained for these regions by considering the sequential transition of earthquakes (Table 1). Selecting one year as a time unit, holding time mass functions $T(m)$ were obtained by taking into account the time lapse between successive earthquakes (Table 2(1-9)). The largest time interval for the region to region transitions was found to be 9 years. Interval transition probabilities, $F(n)$, were calculated using Equation (9). They are shown in Table 3 (1-9) and Figure 2.

Magnitude to Magnitude Transitions

In the second stage, magnitude has been chosen as state. Three magnitude states were established in the whole region. These are

$5.5 \le M \le 6.0$ M1 State1
$6.0 \le M \le 6.5$ M2 State2
$6.5 \le M$ M3 State3

The transition matrix of magnitude to magnitude transitions is given in Table 4. From this and holding time mass functions (Table 5(1-9)), the interval transition probabilities $F(n)$ have been determined (Table 6(1-9) and Figure 3).

After having computed region to region and magnitude to magnitude interval transition probabilities, if we may assume these to be independent for the same time interval, the magnitudes of possible earthquakes in each region can be determined from joint probabilities. For example, Istanbul, the city where we live, is located in region A. The probability of occurrence of an earthquake with $M \ge 5.5$ in an one-year interval in region A, can be estimated as 67% when interval transition probabilities of region to region transitions are evaluated (Fig. 2). The probability of occurrence of a second earthquake with $(5.5 \le M < 6.0)$ *M1* magnitude is 75% for the whole

Table 1. Transition probability matrix of region to region transitions

$$G = [G_{ij}] = \begin{bmatrix} 0.41 & 0.32 & 0.27 \\ 0.25 & 0.50 & 0.25 \\ 0.29 & 0.17 & 0.54 \end{bmatrix}$$

Table 2(1-9). Holding time mass functions of region to region transitions.

$$T(1)=\begin{bmatrix} 0.56 & 0.58 & 0.50 \\ 0.00 & 1.00 & 0.50 \\ 0.72 & 0.50 & 0.77 \end{bmatrix} \qquad T(2)=\begin{bmatrix} 0.11 & 0.14 & 0.17 \\ 0.33 & 0.00 & 0.00 \\ 0.00 & 0.25 & 0.15 \end{bmatrix}$$

$$T(3)=\begin{bmatrix} 0.11 & 0.14 & 0.00 \\ 0.33 & 0.00 & 0.33 \\ 0.14 & 0.25 & 0.00 \end{bmatrix} \qquad T(4)=\begin{bmatrix} 0.00 & 0.14 & 0.17 \\ 0.17 & 0.00 & 0.17 \\ 0.14 & 0.00 & 0.08 \end{bmatrix}$$

$$T(5)=\begin{bmatrix} 0.11 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix} \qquad T(6)=\begin{bmatrix} 0.11 & 0.00 & 0.00 \\ 0.17 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

$$T(7)=\begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix} \qquad T(8)=\begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

$$T(9)=\begin{bmatrix} 0.00 & 0.00 & 0.16 \\ 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

Table 3(1-9). Interval transition probability matrices of region to region transitions.

$$F(3) = \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.23 & 0.53 & 0.24 \\ 0.28 & 0.28 & 0.44 \end{bmatrix} \qquad F(4) = \begin{bmatrix} 0.40 & 0.35 & 0.25 \\ 0.31 & 0.38 & 0.31 \\ 0.33 & 0.32 & 0.35 \end{bmatrix}$$

$$F(5) = \begin{bmatrix} 0.41 & 0.33 & 0.26 \\ 0.34 & 0.35 & 0.31 \\ 0.35 & 0.33 & 0.32 \end{bmatrix} \qquad F(6) = \begin{bmatrix} 0.41 & 0.33 & 0.26 \\ 0.38 & 0.32 & 0.30 \\ 0.36 & 0.33 & 0.31 \end{bmatrix}$$

$$F(7) = \begin{bmatrix} 0.42 & 0.31 & 0.27 \\ 0.40 & 0.31 & 0.29 \\ 0.37 & 0.33 & 0.30 \end{bmatrix} \qquad F(8) = \begin{bmatrix} 0.41 & 0.31 & 0.28 \\ 0.40 & 0.32 & 0.28 \\ 0.38 & 0.33 & 0.29 \end{bmatrix}$$

$$F(9) = \begin{bmatrix} 0.38 & 0.31 & 0.31 \\ 0.40 & 0.32 & 0.28 \\ 0.39 & 0.32 & 0.29 \end{bmatrix}$$

Table 4. Transition probability matrix of magnitude to magnitude transitions.

$$G = [G_{ij}] = \begin{bmatrix} 0.63 & 0.17 & 0.20 \\ 0.54 & 0.15 & 0.31 \\ 0.56 & 0.19 & 0.25 \end{bmatrix}$$

Table 5(1-9). Holding time mass functions of magnitude to magnitude transitions.

$$T(1)=\begin{bmatrix} 0.65 & 0.43 & 0.88 \\ 0.43 & 1.00 & 0.00 \\ 0.78 & 1.00 & 0.50 \end{bmatrix} \qquad T(2)=\begin{bmatrix} 0.07 & 0.15 & 0.12 \\ 0.15 & 0.00 & 0.25 \\ 0.00 & 0.00 & 0.50 \end{bmatrix}$$

$$T(3)=\begin{bmatrix} 0.12 & 0.14 & 0.00 \\ 0.14 & 0.00 & 0.25 \\ 0.22 & 0.00 & 0.00 \end{bmatrix} \qquad T(4)=\begin{bmatrix} 0.12 & 0.14 & 0.00 \\ 0.14 & 0.00 & 0.25 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

$$T(5)=\begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.14 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix} \qquad T(6)=\begin{bmatrix} 0.04 & 0.14 & 0.00 \\ 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

$$T(7)=\begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix} \qquad T(8)=\begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

$$T(9)=\begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.25 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

Table 6(1-9). Interval transition probability matrices of magnitude to magnitude transitions.

$$F(1) = \begin{bmatrix} 0.75 & 0.07 & 0.18 \\ 0.23 & 0.77 & 0.00 \\ 0.44 & 0.19 & 0.37 \end{bmatrix} \qquad F(2) = \begin{bmatrix} 0.69 & 0.15 & 0.16 \\ 0.29 & 0.59 & 0.12 \\ 0.43 & 0.20 & 0.37 \end{bmatrix}$$

$$F(3) = \begin{bmatrix} 0.66 & 0.19 & 0.15 \\ 0.37 & 0.45 & 0.18 \\ 0.59 & 0.22 & 0.19 \end{bmatrix} \qquad F(4) = \begin{bmatrix} 0.64 & 0.22 & 0.14 \\ 0.46 & 0.32 & 0.22 \\ 0.58 & 0.23 & 0.19 \end{bmatrix}$$

$$F(5) = \begin{bmatrix} 0.62 & 0.22 & 0.16 \\ 0.57 & 0.25 & 0.18 \\ 0.60 & 0.23 & 0.17 \end{bmatrix} \qquad F(6) = \begin{bmatrix} 0.60 & 0.23 & 0.17 \\ 0.54 & 0.28 & 0.18 \\ 0.61 & 0.22 & 0.17 \end{bmatrix}$$

$$F(7) = \begin{bmatrix} 0.59 & 0.23 & 0.18 \\ 0.55 & 0.28 & 0.17 \\ 0.60 & 0.23 & 0.17 \end{bmatrix} \qquad F(8) = \begin{bmatrix} 0.59 & 0.23 & 0.18 \\ 0.56 & 0.29 & 0.15 \\ 0.59 & 0.24 & 0.17 \end{bmatrix}$$

$$F(9) = \begin{bmatrix} 0.59 & 0.23 & 0.18 \\ 0.54 & 0.22 & 0.24 \\ 0.59 & 0.24 & 0.17 \end{bmatrix}$$

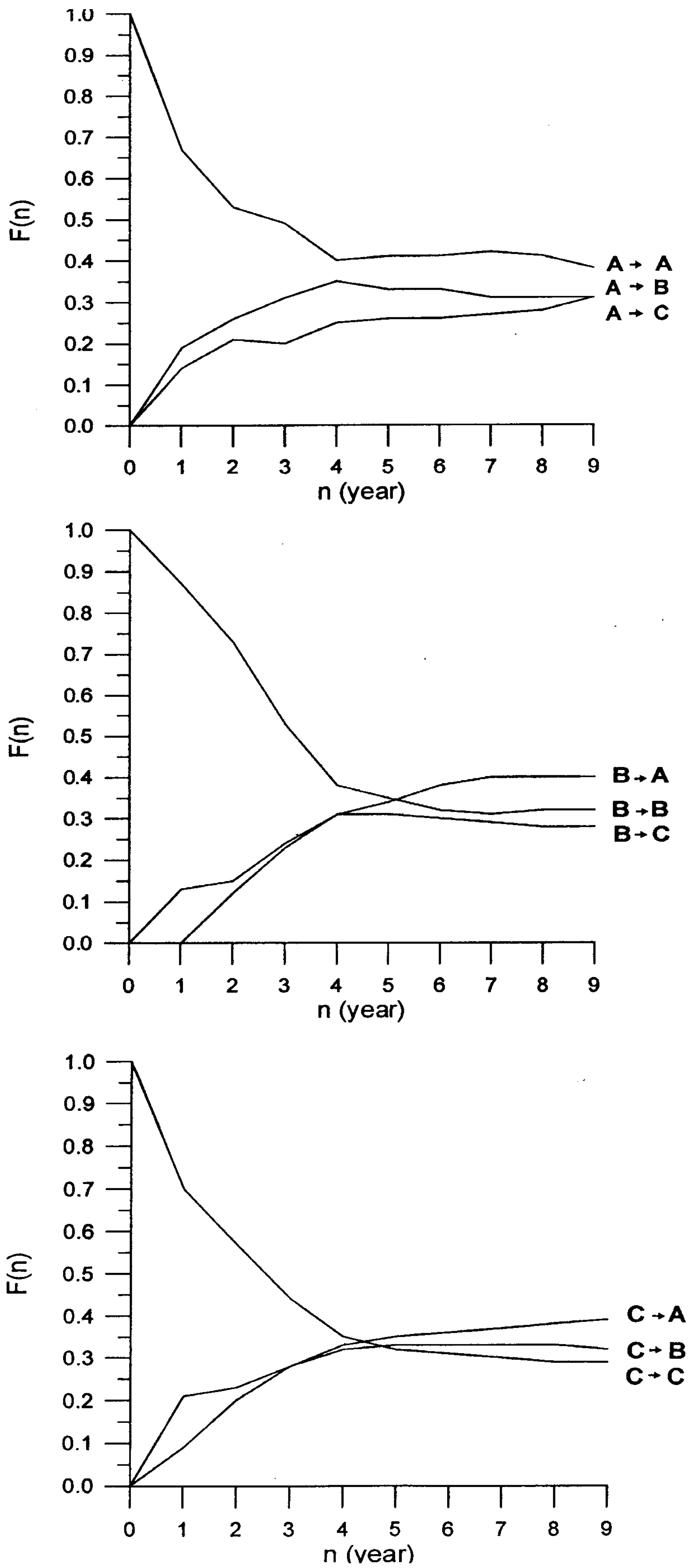

FIG. 2. Interval transition probability functions for region to region.

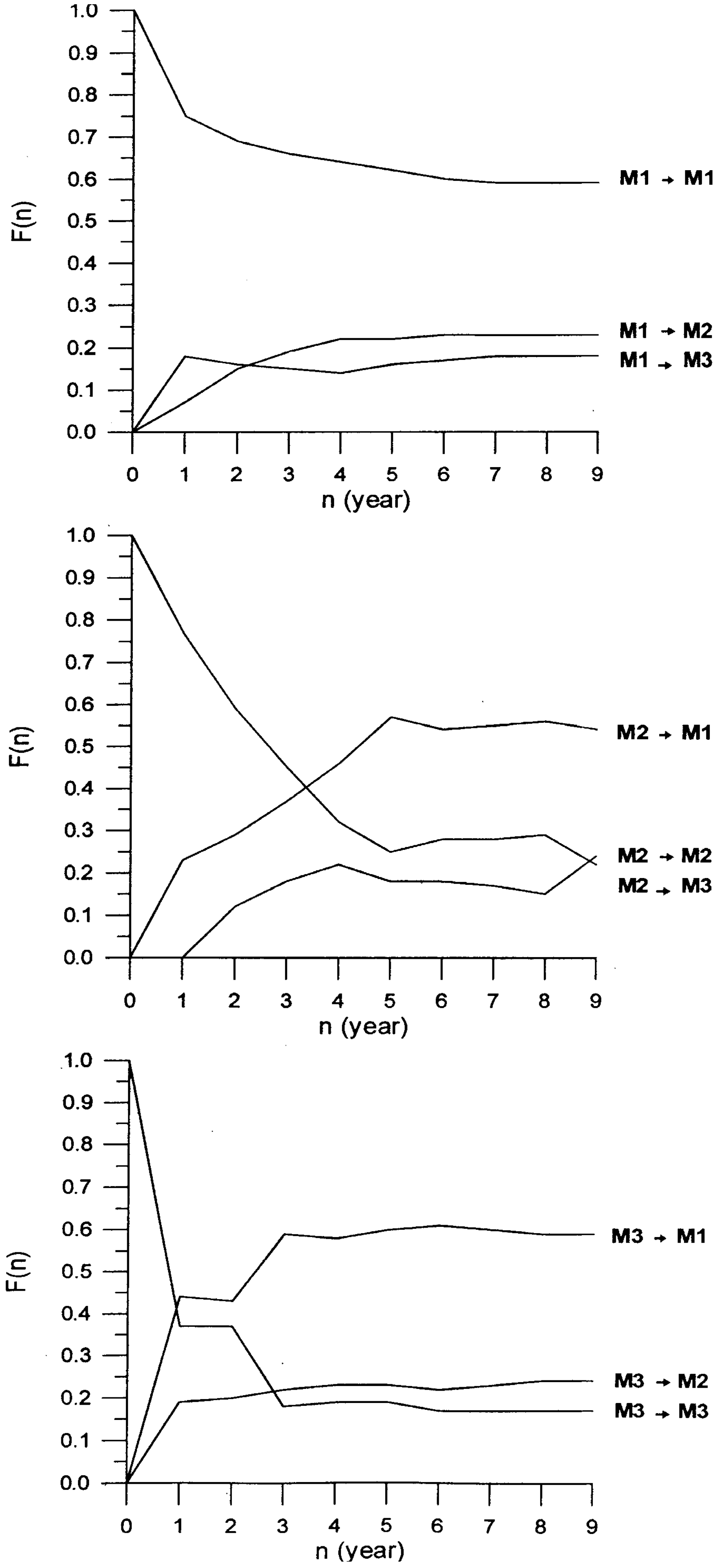

FIG. 3. Interval transition probability functions for magnitude to magnitude.

region according to the interval transition probabilities of magnitude to magnitude transitions (Fig. 3). The joint probability value has been determined as 50%. This value represents the occurrence probability of an earthquake of *M1* magnitude that follows an earthquake of the same magnitude in an one-year interval for the region A. Similarly, joint probability values of three magnitude states in each region can be assessed according to the time interval between the successive events. The recent 17 August 1999 and the previous 15 March 1992 earthquakes were in the regions C and A, respectively. Figure 2 shows that the interval transition probabilities of region to region for seven years, the probability of C to A transition is relatively higher than that of the others.

CONCLUSION

In this study, the earthquake occurrence probabilities have been obtained for the North Anatolian Fault Zone by applying the semi-Markov model. It can be deduced from the interval transition probabilities of the region to same region, every region shows the similar features following $M \geq 5.5$ earthquakes. The probability of an earthquake of the same magnitude decreases as the time interval increases, while other region to region transition probability increases with time.

According to the magnitude to magnitude interval transition probabilities, the occurrence probability of an earthquake of equal magnitude following one of ($5.5 \leq M1 < 6.0$) generally decreases while the occurrence probability of *M1* magnitude earthquake coming after ($6.0 \leq M < 6.5$) *M2* and ($6.5 \leq M$) *M3* increases with time.

From joint probabilities obtained with the cumulative evaluation of region to region and magnitude to magnitude transitions, the probability of occurrence of an earthquake of *M1* magnitude following an earthquake of the same magnitude in a one-year interval is considerably high for every region. Thus, three-dimensional evaluation in terms of space, time and magnitude has been possible. The maximum time interval considered in the model is nearly equal to the observed maximum time interval. This is due to the fact that no specific distribution function has been chosen. However, this model may provide valuable information for earthquake occurrences if the method is applied to regions of uniquely defined tectonic features.

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