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Exploratory analysis of marked poisson processes applied to Balkan earthquake sequences

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***Abstract**: We present some applications of marked Poisson processes to analyse catalog data of earthquakes. A compound Poisson process has been used to model cumulative energy release of main shocks in the Balkan region. Also, a marked Poisson process has been applied to model number of events at different magnitude levels. Another model has been developed to feature the joint distribution of interoccurrence times and corresponding magnitude differences between subsequent events. Results point out that marking of the process is a helpful instrument, enabling us to catch some features of the physical process underlying the data.*

***Keywords**: Marked Poisson Processes, Earthquakes Sequences, Models of Earthquake Catalog Data.*

INTRODUCTION

Earthquakes could be regarded as discrete events, representing some real, not well known tectonic process. Following that scheme and having in mind the highly random character of all earthquake parameters, it is quite natural to consider a sequence of earthquakes as a stochastic process and more explicitly as a point stochastic process.

In most cases, when studying earthquake occurrence as a stochastic process, only times of events are considered. As they seem to occur randomly in time, the main aim has most often been to test whether real data support such an assumption. In the theory of stochastic processes Poisson processes are models of phenomena, which exhibit highly random behavior. When only occurrence times $\{T_i : i=1, 2 ,\dots ,n\}$ are considered, simple Poisson process is used as a model of randomness, to which real data are compared.

It has always been of interest to try to enrich time models of earthquakes with information about other event's parameters i.e. to develop space-time models or the ones relating occurrence times with a quantity, representing the size of an event (magnitude, intensity or energy); or to incorporate in the models some deviations from the Poisson process. In more recent years point processes have been used to include in the models aftershock occurrence (Ogata, 1988).

The aim of this study will be, given a sequence of seismic events, to search for some relations between occurrence times $\{T_i : i=1, 2, \dots, n\}$ and some quantity $\{W_i : i=1, 2, \dots, n\}$, giving information about size of the events under study. Such relations could then be incorporated in stochastic modeling of the sequence. The study will be done for main events mostly ; they indeed play major part in estimating seismic risk. More often it is assumed that there is independence between the time T_i of a main event and its size W_i, and also among the values of W_i. These will be our initial assumptions, too.

The aim of our study will be to develop a model process, implying these assumptions, and to verify how well it fits the data set.

MARKED POISSON PROCESS MODELS

There is an analog of a counting process for marked point processes. It is sometimes termed as a mark-accumulator process and is defined as follows.

Let's have a Poisson process $\{N(t) : t \geq 0\}$ with a rate $\lambda > 0$ and suppose that the time T_i of each event is associated to a realization of a random variable Y_i, where $\{Y_n : n > 0\}$ is a family of independent and identically distributed random variables sharing the distribution

$$G(y) = \Pr\{Y_k \leq y\} \quad (1)$$

A second requirement is that they be independent of $\{N(t): t \geq 0\}$, too. Then the stochastic process

$$Z(t) = \sum_{k=1}^{N(t)} Y_k \qquad \text{for } t \geq 0 \tag{2}$$

is said to be a compound Poisson process (Taylor and Karlin, 1984; Ross, 1980)

If $\mu = E[Y_1]$ and $v^2 = Var[Y_1]$ are the common mean and variance for $Y_1, Y_2, ...,$ then the moments of $Z(t)$ are given by:

$$E[Z(t)] = \lambda \mu t \tag{3}$$

$$Var[Z(t)] = \lambda(v^2 + \mu^2)\, t \tag{4}$$

As can be seen from the definition of compound Poisson process, it implies both assumptions made by us for main shocks and it could be used as a model process of random behavior in case a sequence of occurrence times and size of events is to be analyzed.

In the most general treatment of marked point processes $\{N(t) : t \geq 0\}$ is an inhomogeneous Poisson process with an intensity function $\{\lambda(t) : t \geq 0\}$ and the marks $\{Y_n : n>0\}$ need not form an independent sequence of random variables. Nor it is required that the marks be independent of the counting process or the occurrence time sequence (Snyder and Miller, 1991).

If we restrict ourselves to consider an inhomogeneous compound Poisson process with a rate $\lambda=\lambda(t)$, then formula (3) and (4) would be translated into

$$E[Z(t)] = \mu \int_{t_o}^{t} \lambda(s) ds \tag{5}$$

$$Var[Z(t)] = (v^2 + \mu) \int_{t_0}^{t} \lambda(s) ds \tag{6}$$

following (Snyder and Miller, 1991). The problem of obtaining a proper model of $\lambda=\lambda(t)$ is difficult enough itself and for the time being, however, we shall follow the initial assumptions and refer to a stationary Poisson process.

If we put our attention on the ways the size of an earthquake can be represented, we can consider the following reasoning. A natural way to estimate the size of an event is by the energy of the propagated elastic waves. This approach, however, has some restrictions, firstly in the way the energy is determined from the instrumentation record of an earthquake and secondly, in the fact that the dynamic range of energies released is very broad, which causes some computational difficulties. It is convenient, however, to consider the sequence of released energies in the scheme of a compound Poisson process, as the cumulative energy, released, is a real physical quantity.

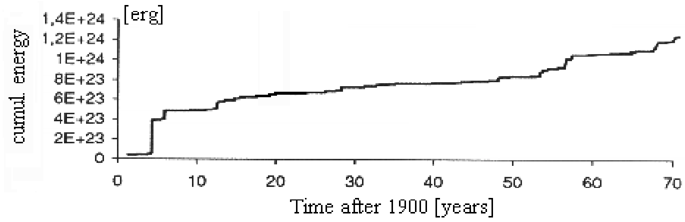

FIG. 1. Released cumulative energy for the earthquakes in the Balkan region in the period 1900-1970 with aftershocks excluded, $M_{LH} \geq 5.0$

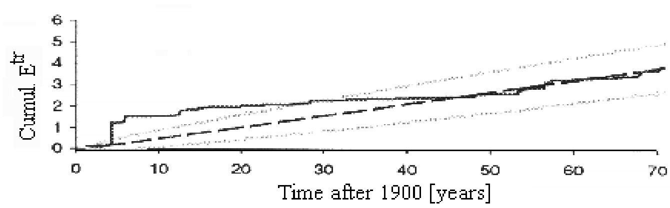

FIG. 2. Cumulative transformed energy (solid line) and estimated compound process (dashed line) for the Balkan catalog with aftershocks excluded, $M_{LH} \geq 5.0$;for energy transformation see formula (8)

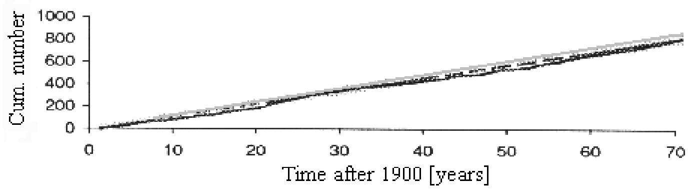

FIG. 3. Cumulative number of events (solid line) and estimated simple Poisson process (dashed line). for the Balkan catalog with aftershocks excluded, $M_{LH} \geq 5.0$

Another way to express the size of an event is by the maximum intensity I_o at the epicenter. For the present study, we consider it not very suitable to be used as a mark of the process.

The most frequent way to represent the strength of an earthquake is by its magnitude; as it is a logarithmic function of the released elastic energy (7), the range in which magnitudes vary is less large and this makes them more suitable for computational use. In case we consider a compound Poisson process, however, it is more reasonable to use energy, because a sum of magnitudes is a quantity rather faint from a physical point of view.

In the following we have chosen the energy of an event as measure of its strength and we have analyzed the data in the frame of a compound Poisson process. To calculate the energy given the magnitude we have used the formula (Gutenberg, 1956; Jarkov, 1983)

$$\lg E_s = 11.8 + 1.5 M_s \tag{7}$$

To reduce, however, the very broad variability range of the energy released in an earthquake, we have transformed this quantity through the formula

$$E_i^{tr} = \frac{(E_i - E_{\min})}{(E_{\max} - E_{\min})} \quad ; \qquad E_i^{tr} \in [0,1] \qquad (8)$$

where E_{max} and E_{min} are respectively the upper and lower bound of the observed values, and E_i is the energy of the *i*-th event.

As can be seen comparing Figure 1 and Figure 2, this parameter reflects the behavior of real energy enough well and is much more suitable for computational use at the same time.

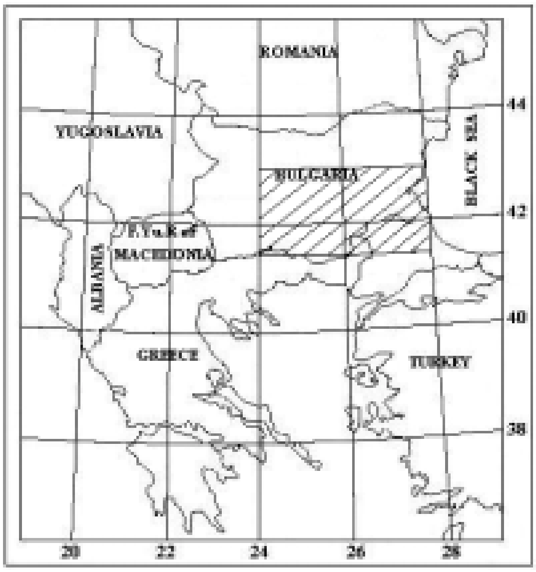

FIG. 4. Map of the area covered by the Balkan catalog. Dashed area is a seismic zone in Bulgaria, specified in text. Seismic sequences in that zone have been modeled by a compound Poisson process and results are being presented in Figs.5-8.

The approach, described above, will first be tested on data from the Balkan catalog. At the beginning, the whole Balkan catalog will be considered with no aftershocks, so as to exclude already known deviations from our initial assumptions. The Balkan catalog contains more than 4000 earthquakes, covering the period 1900-1970, but with aftershocks excluded, the number is reduced to about 800. The identification of such aftershocks has been done in an earlier study (Gospodinov, 1990), following the algorithm known as Knopoff's window. Results, obtained by Karnik and Prochazkova (1973), point out that a lower cut-off limit of M_{LH} = *5.0* is suitable, beneath which the catalog suffers certain lack of events.

The Balkan catalog covers quite a large area, including different seismic zones. Considering the whole catalog could possibly destroy some peculiarities of the seismic process in each zone, but, on the other hand, that will make possible deviations from the assumptions of randomness even more important.

In Figure 2 we have plotted (dashed line) the estimated mean of the compound Poisson process

$$Z(t) = \sum_{i}^{N(t)} E_i^{tr} \qquad (9)$$

following formula (2) and the standard deviation used as confidence interval bounds. We have also plotted the real cumulative E^{tr} process. As can be seen, a great portion of energy, released at the beginning of the period, causes an essential difference between the real process and the compound Poisson process for some time. Unfortunately, the period covered by the data is too short and we have only this case of deviation of the real process from the estimated one, but one of the possible interpretations of the results could be the following.

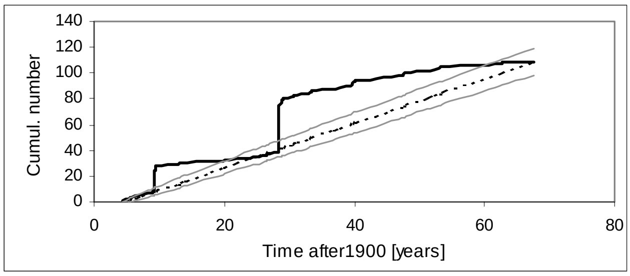

FIG. 5. Cumulative number of events (solid line) and estimated Poisson process (dashed line) for the seismic zone in Bulgaria, specified in Figure 4.

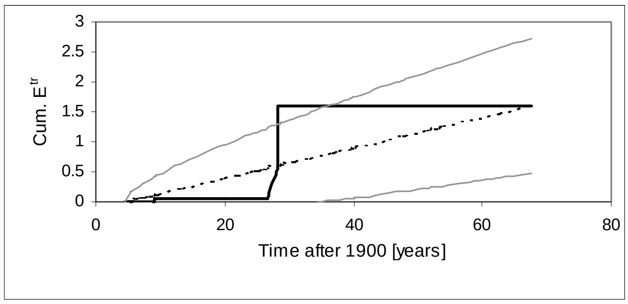

FIG. 6. Cumulative transformed energy (solid line) and estimated compound Poisson process (dashed line) for the seismic zone in Bulgaria, specified in Figure 4.

If we consider the Balkan region to be subjected to tectonic processes, leading to a constant accumulation of stress, we may suppose that it reaches some critical state, which determines random (for the whole region) release of stress in different seismic zones giving rise to earthquakes with different energy.

But the release, owing to one or a few very strong earthquakes, of a very big portion of accumulated stress can cause considerable departure from the state of criticality, which reflects in deviation from randomness in the earthquake occurrence, too. Then the process of constant stress accumulation will take some time for the region to return to the state of criticality and for the process of energy release to return to randomness. It could be very interesting to test such an interpretation on different data sets for longer period of time.

In Figure 3 we have plotted the estimated mean and corresponding error bounds for a simple Poisson process. We have also plotted the cumulative number of events in time. There are some small deviations of the real process from the Poisson one, but they do not seem to be essential. This figure is not very informative except for the fact that the counting process is well fitted by a simple Poisson process. Put from the viewpoint of hazard evaluation we are interested not only in the number of future events but also, and probably mainly, in their strength.

If we compare Figure 2 with Figure 3, it can be seen, that marking of the process is an useful tool to extract more information from the data about the earthquake process, as the marked process shows deviation from randomness, which should further be studied and interpreted.

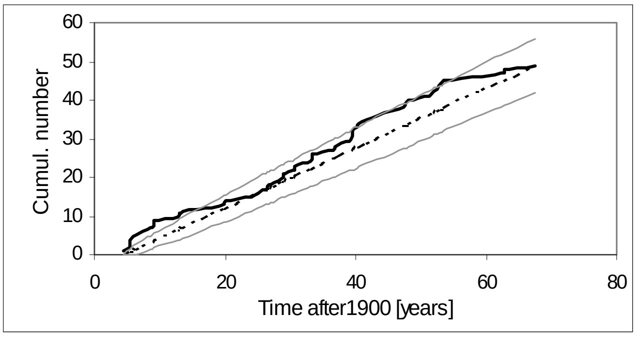

FIG. 7. Cumulative number of events (solid line) and estimated Poisson process (dashed line) for the seismic zone in Bulgaria , specified in Figure 4 with two main groups of aftershocks in 1909 and 1928 excluded.

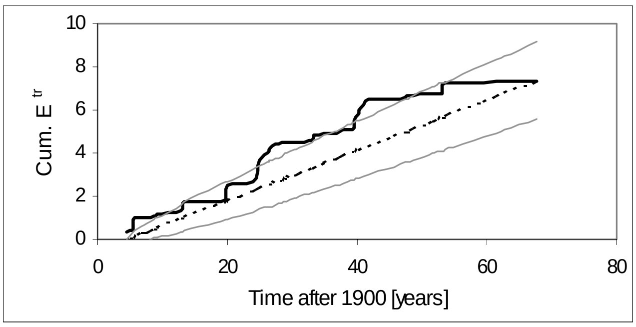

FIG. 8. Cumulative transformed energy (solid line) and estimated compound Poisson process (dashed line), concerning the case in Figure 6.

Non-random behavior, that is, some kind of relation between occurrence time and released energy is more expected to be identified for a smaller, more homogeneous seismic zone. Such a zone has been chosen in Bulgaria; it has comparatively high seismicity and several strong events have occurred in it. Geographically, it can be described as a region with vertices, located by the following latitudes and longitudes and is shown in Figure 4 (dashed area)

41.5N, 24.0E	43.0N, 28.0E
43.0N, 24.0E	41.5N, 28.0E

Here all the events with magnitudes $M_{LH} \geq 4.0$ have been considered, whose number is $N=111$, since the number of events with $M_{LH} \geq 5.0$ is too small.

This, however, as shown in Figure 5, leads to the fact, that one of our initial conditions is not satisfied because the examined data set includes aftershocks and the distribution of events in time is not simple Poissonian.

As said before, for this case it would be better to use formula (5) and (6), because they consider the case of an inhomogeneous compound Poisson process. But here $\lambda=\lambda(t)$ is not known and only some general features of the seismic process in the zone will be discussed.

Another conclusion, drawn from Figure 6 is that for a small seismic zone, where several strong events have occurred in a short time interval, the mark, defined by formula (8), is not quite suitable. That is because the energy of the weakest event is not comparable with the total energy released in the highly active period.

Both figures hint a peculiar feature of the seismic process in the investigated zone. Excluding two quite short periods (each about one year) characterized by a particular increase of both energy release and number of events, for the rest of the time period, spanned by the catalog, the process seems to be quite near a stationary one.

To check that, both groups of earthquakes (including main shocks) have been removed and Figures 7 and 8 reflect the seismic process for the remaining part of the catalog. As can be seen, the assumptions of stationarity and independence between time and energy are roughly supported by the data in this case.

So, the seismic regime for the investigated zone is characterized by short period deviations from randomness, in which energy release and number of events (mainly aftershocks) are particularly high. After them the process comparatively quickly returns to stationarity.

The problem of a possible relation between occurrence times and size of earthquakes in the catalog could be regarded from another point of view, too. The real seismic process, reflected in the catalog could be considered as a composition of processes at different energy levels.

In the approach, applied above, the observed seismic process is represented through the cumulative energy and is compared to a model, the compound Poisson process. Such an analysis exhibits rather general characteristics of the earthquake process.

Now we shall try to decompose the process at different size levels and follow its behavior, modeling it through a marked Poisson process. Suppose again, that we have a Poisson process $\{N(t), t\geq 0\}$ with a rate λ and the time of each event is associated with a random variable Y_k such that Y_1, Y_2, ... could be considered as independent and sharing the common distribution function, given by formula (1).

Then the sequence of pairs (T_1,Y_1), (T_2,Y_2), ... is called marked Poisson process and forms a two-dimensional, inhomogeneous Poisson point process in the (t, y) plane (Taylor and Karlin, 1984). The mean number of points in set A of the (t, y) plane is given by

$$\mu(A) = \iint_A \lambda g(y)\,dt\,dy \qquad (10)$$

Now if we get back to the initial assumptions of stationarity in time and independence of the mark from the occurrence time, we can use the marked point process, presented by formula (10) as a model process, embodying our assumptions. It then gives the expected number of events for each region A_{ij} in the plane (t, y), to which real data could be compared.

$$\mu(Aij) = \iint_{A_o} \lambda g(m)\,dt\,dm = \int_{m_i}^{m_{i+1}} \{ \int_{t_i}^{t_{i+1}} \lambda dt \} dm \qquad (11)$$

By m here we denote the magnitude of an event, which has been used as a marking parameter in this case. It is known empirically that the magnitudes follow roughly an exponential distribution

$$\Pr(M > m) = \exp(-\beta m) \qquad (12)$$

Then $g(m)$ could be expressed by

$$g(m) = \beta \exp(-\beta m) / [\exp(-\beta M_L) - \exp(-\beta M_H)] \qquad (13)$$

where M_L and M_H are the lower and upper cut-off limits of the magnitudes considered. So, for (A_{ij}) we obtain

$$\mu(A_{ij}) = \lambda(t_{i+1} - t_i) \int_{m_i}^{m_{i+1}} \{\beta \exp(-\beta m)/[\exp(-\beta M_L) - \exp(-\beta M_H)]\} dm = \qquad (14)$$
$$= \lambda(t_{i+1} - t_i)[\exp(-\beta m_i) - \exp(-\beta m_{i+1})]/C$$

where by C we have denoted the difference $[\exp(-\beta M_L)- - \exp(-\beta M_H)]$.

It might be questionable here, how useful such a decomposition of the process is from a physical point of view. Getting back to the idea, that earthquake occurrences in a seismic zone reflect some more general tectonic process, it could be reasonable to assume that seismic energy release at different levels is interrelated. Up to now, however, excluding clustered events, there are no dependencies, identified between events with different magnitudes in a seismic zone.

What is more, speaking in terms of a seismic cycle (Sobolev, 1995), there are some results from laboratory experiments concerning rock failure, according to which energy release differs in time for different levels. There are also some results, showing the existence of a quiescence period before strong earthquakes (Mogi, 1968; Fedotov, 1968). They point out, that this quiescence is connected with background seismicity of 2 to 3 units weaker than the impending earthquake, speaking in terms of magnitude.

That explains why modeling of the seismic process by a marked point process and its decomposition could turn out to be a useful approach. This methodology has again been tested on a sample of the Balkan catalog, containing earthquakes with $M_{LH} \geq 5.0$ and deprived of aftershocks. The β parameter has been estimated using a maximum likelihood procedure which has yielded $\beta = 2.19$.

The results are plotted in Figure 9. Here, four magnitude intervals are considered and the time interval chosen is three years. The magnitude ranges were chosen so, that we could obtain sufficient number of cases for each cell in the (t, m) plane. The curves represent the difference between the real and expected frequency. The occurrence times of strong events with $M_{LH} \geq 7.0$ are also shown.

As can be seen, the processes at different energy levels are similar and the marked point process roughly fits the data. There are some peculiarities, however, pointed out by Figure 9. For the magnitude range *5.0 - 5.1* we have an exceedance of the real frequencies compared to the expected ones (nearly all the plotted values are positive); this fact could be due to an underestimate of β.

Some long-term trends of the *5.0 - 5.1* level can also be identified. These trends and, more specifically, the minimum in the *5.0 - 5.1* curve could be linked to the lack of events with $M_{LH} \geq 7.0$ in that period. This result shows, that marking of the process could successfully be applied to identify quiescence periods before strong events.

Of course, we understand, that the possible interpretations of these results are quite general and this method could be much more useful if applied to a smaller, more homogeneous seismic zone. This, however, is not feasible because the model would require larger samples than the available ones in such zones.

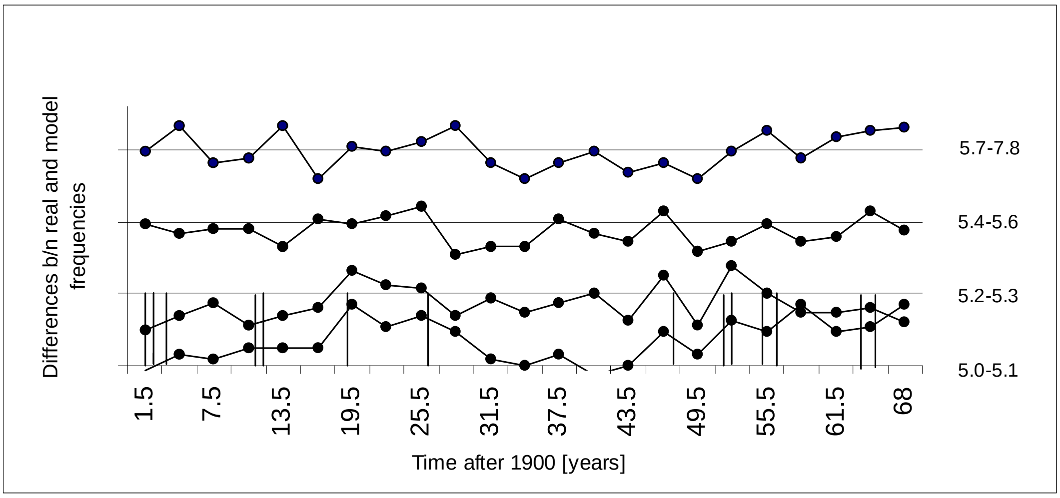

FIG. 9. Differences between real and model frequencies for four magnitude ranges (denoted to the right) and for time intervals of three years. Each value is for different cells in the *(t, m)* plane for the Balkan catalog with aftershocks removed ($M_{LH} \geq 5.0$). Vertical lines denote occurrences of earthquakes with $M_{LH} \geq 7.0$

(ΔT,ΔM) MODEL

We suggest another approach to the problem of identifying possible relationships between occurrence times and earthquake magnitudes. If we consider the differences between the magnitudes of subsequent earthquakes and the corresponding interoccurrence times, we could incorporate their independence in a model and check how well it fits the data.

Let us have a sequence of *N+1* events. From this sequence we obtain N differences ΔM between subsequent magnitudes and N corresponding interoccurrence times *ΔT*. Assuming the number of occurrences in time follows the Poisson distribution and ΔT and ΔM are independent each other, we can obtain their joint distribution and use it as model of our data.

$$F(\tau,\eta) = \Pr(\Delta T < \tau, \Delta M < \eta) = \Pr(\Delta T < \tau)\Pr(\Delta M < \eta) \quad (15)$$

What we need, to obtain the explicit form of formula (15) are the distribution functions of *ΔM* and *ΔT*. The exponential distribution of *ΔT* follows easily from the Poisson process assumption, whereas we could develop the distribution function of *ΔM*, basing on the recurrence law and the general assumption that main shocks have independent magnitudes.

If we denote

$$Z = M_i - M_{i-1} = Y - X \qquad i = 2, 3, \ldots, N+1$$

following formula (13), then we reduce the problem to finding the distribution of the difference of two independent, identically distributed random variables (Blom, 1989) with

$$g(x) = \beta \exp(-\beta x)/C \qquad X \in A = [M_L, M_H]$$

and

$$Z = Y - X \qquad Z \in [M_L - M_H, M_H - M_L]$$

$$F(z) = \Pr(Z < z) = \iint_B f(x,y)dxdy = \iint_B g(x)g(y)dxdy \quad (16)$$

where by C we have again denoted [exp(-βM_L) - - exp(-βM_H)] and by $B = \{(x,y) \in A \times A\colon y - x < z\}$. We have solved the problem in two phases;

(i) $z \in [M_L - M_H, 0]$

$B = \{(x, y) : x \in [M_L - z, M_H], y \in [M_L, z+x]\}$

$$F(z) = \int_{M_L - z}^{M_H} [\int_{M_L}^{z+x} \frac{\beta \exp(-\beta y)}{C} dy] \frac{\beta \exp(-\beta x)}{C} dx = \quad (17a)$$

$$= \{0.5\exp[-\beta(2M_L - z)] - \exp[-\beta(M_H + M_L)] + 0.5\exp[-\beta(2M_H + z)]\}/C^2$$

(ii) $z \in [0, M_H - M_L]$

$B = \{(x, y) : x \in [M_L, M_H - z], y \in [z+x, M_H]\}$

$$F(z) = 1 - \Pr(Z > z) = 1 - \iint_B g(x)g(y)dxdy$$

$$= 1 - \int_{M_L}^{M_H - z} [\int_{z+x}^{M_H} \frac{\beta \exp(-\beta y)}{C} dy] \frac{\beta \exp(-\beta x)}{C} dx \quad (17b)$$

$$= 1 - \{0.5\exp[-\beta(2M_L + z)] + 0.5\exp[-\beta(2M_H - z)] - \exp[-\beta(2M_L + M_H)]\}/C^2$$

The obtained distribution function and its density are represented in Figure 10. We can now get back to formula (15) and, substitute the probability of *ΔT* by an appropriate exponential distribution and the probability of *ΔM* by eqs. 17a-b. In this way we have specified the joint distribution of a model of the magnitude difference and the interevent time of subsequent events. We then compared this model with real data. In Figure 11 we have parted the *(ΔT, ΔM)* plane into cells and there we have plotted the normalized (divided to the number of all cases) differences between real and model frequencies for each cell. The size of the cells has again been chosen so, that the number of observations in each cell is not too small.

As we have not done any specific alternative hypothesis to the null hypothesis of independence of *ΔT, ΔM* (see eq.15), we have used the $\aleph^2$ - test to verify the goodness of fit of our model to the data. The test rejects H_0 with p=0.01 significance level, hence the independence model does not show a good fit to the data.

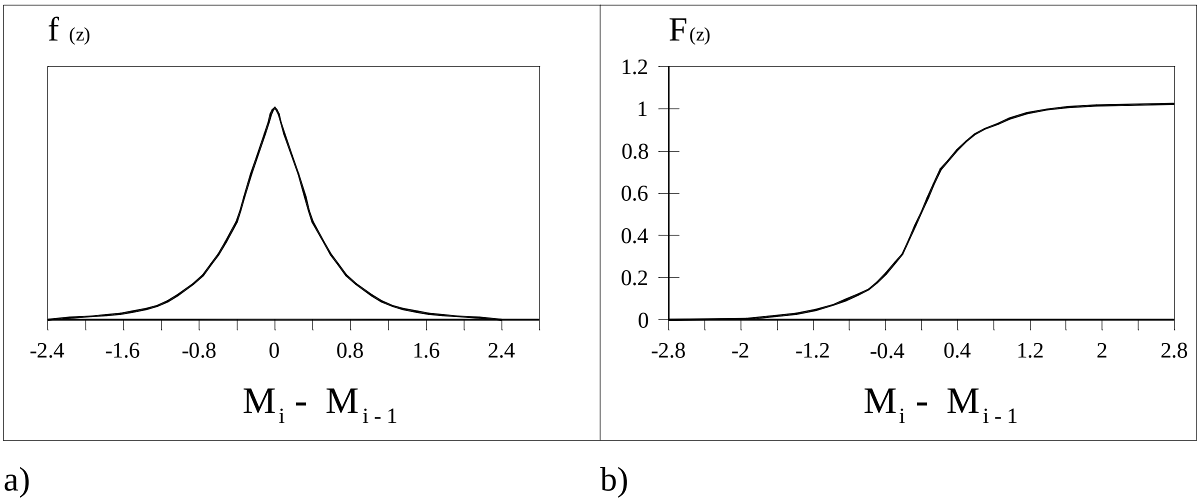

FIG. 10. a) Density function of magnitude difference $\Delta M = M_i - M_{i-1}$; b) Distribution function of $\Delta M = M_i - M_{i-1}$

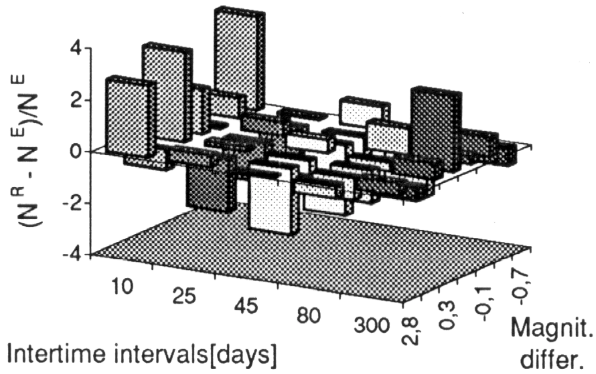

FIG. 11. Differences between real and model frequencies for cells in the *ΔT, ΔM* plane for the Balkan catalog, MLH ≥ 5.0 (N^E - expected, N^R - real). Model distribution is calculated on the base of eq. 15 (see in text) in which *ΔT* follows an exponential distribution and *ΔM* follows the distribution given by eqs.17a-b

It can be seen, that there are some essential discrepancies between the observed data set and both univariate distributions of *ΔM and ΔT*, which points to deviations from our assumptions of independence between subsequent magnitudes and of Poisson occurrences in time. The exceedance of real frequencies for nearly all the short intertime cells can be connected to the existence of grouping of main shocks. Similar results for this set of data have earlier been obtained in (Gospodinov, 1990).

CONCLUSIONS

Our purpose in this study has been to verify the applicability of some marked point processes as models of our data, so as to obtain more information about possible relations between occurrence times of earthquakes and their sizes.

Two stationary marked Poisson processes have been applied to analyse catalog data : a compound Poisson process has been used to model the behavior of the cumulative energy release and a marked Poisson process - to model the behavior of the magnitude at different levels.

Both of them imply assumptions of independence between the mark realizations and independence of the mark from the occurrence time. These assumptions are very restrictive, but in the case of main shocks they seem to be natural.

Another model has been developed to consider the joint distribution of the interoccurrence times and of the corresponding magnitudes.

The results, obtained by the marked Poisson process model *(ΔT, ΔM),* point out the existence of some deviation from the assumption of independence between mark and occurrence time. A very large portion of energy release could possibly lead to a comparatively long period of low energy release rate. An effect of main shock grouping has also been identified, which, in our opinion, deserves special attention, as it is quite important for seismic risk assessment.

The application of marked point processes to study the earthquake process from the viewpoint of possible occurrence time - size of an event relation, has revealed some peculiar features :

(i) the first of them is connected with the mark itself; on one hand the usual way of estimating the size of an event by its magnitude is not suitable for a mark-accumulator process and, on the other hand, the energy has a too broad range, which leads to some computational difficulties and problems in the graphical representation, too;

(ii) addition of a mark to the occurrence time process leads to an augmentation of sample size needed to get reliable results. That causes some difficulties in the use of marked point processes for small, more homogeneous seismic zones, to which they could more successfully be applied.

Comparing the stochastic processes we applied to model seismic catalog data - simple Poisson process, marked Poisson process and compound Poisson process, we could infer the following: simple Poisson process is the classical model of main shock occurrences when only temporal behavior is studied and it is not adequate if we want to incorporate more information about the seismic process. For that purpose the compound Poisson process is more suitable. By definition this process could be used for parameters (marks) of earthquake occurrence for which their cumulative value is physically defined (as energy). Finally if we want to decompose the process of earthquake occurrence at different levels of the studied parameter (mark), a marked Poisson process should be the chosen to model our data.

On the whole, marking of the seismic process has turned out to be an useful tool for the cases considered; it has enabled us to get more information about the process from physical point of view, information which could lead us to develop an enriched model afterwards.

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