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# Estimation of some bodies parameters from the self potential method using Hilbert transform

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***Abstract:*** *Structural parameters can be determined directly from the geophysical anomalies having two-dimensional distributions for the potential fields. In order to achieve this, more equations are constructed for the same parameter to be determined. The special properties of analytical functions of the complex gradients and the Hilbert Transforms can be used to reach the above-mentioned situation. Certain structural parameters of two-dimensional bodies such as horizontal cylinder and horizontal sheets, were directly determined from the anomalies of the self-potential method. Hilbert Transforms, which can be carried out in two different ways using the Fourier Transform and convolution methods, were used to provide the conversion between the complex gradients of the anomaly. Structural parameters were then determined from the solutions of the constructed equations (anomaly, amplitude and phase functions).*

***Key Words****: Self Potential, Inversion, 2D Bodies, Hilbert Transformation.*

## INTRODUCTION

Although Hilbert Transform (HT) have been used in electrical engineering and signal analysis for a long time **(**Bracewell, 1965**)**, their application in geophysical studies started following 1970's. The HT is a method of direct solution. The aim of using the HT in geophysical studies is to obtain more than one equations that contain the same structural parameters by utilizing the complex gradients of the available data. The roots and common intersection points of the anomaly and the complex gradients of the anomaly have been used to determine the structural parameters. Therefore, an error ±1 sampling interval is expected for the determinations. In order to minimize the error, the most appropriate sampling interval should be chosen.

The parameter solution equations are different for every structure. Therefore, the structural models belonging to the anomaly should be identified before the application of HT. The HT application can be carried out through the applications of Fourier Transform (FT) and convolution methods. However, in cases where the HT is obtained through FT, the discontinuity that might occur at the terminals of the anomaly causes errors in the determination of the roots and common transaction points. In order to avoid this, this discontinuity should be eliminated before application of HT to the anomaly. The shifts occur also for the roots and common transaction points depending on the length of the convolution operator in the same way. Therefore, one should be careful for the selection of the convolution operator length.

As known from signal analysis, the signals should be identified at definite intervals or should be zero or asymptotic to zero beyond this interval. The data, which are not asymptotic to zero or not zero beyond the identification interval, were screened by using certain procedures (i.e. base reduction, windowing or derivation procedures) in order to obey the above mentioned conditions. Additionally, the equations were simplified for the structures having the **logarithmic** and **arctan** components, through the derivation operation with respect to x before the application of HT for the sake of procedural simplicity.

HT, by definition, is a mathematical transform function, which shifts the phase of a signal by 90° without changing its amplitude. In other words, HT is a linear system, which transforms the odd and even functions, with the same amplitude, into each other in a space or frequency domain.

HT were primarily used to investigate structural parameters in magnetic method by applying the magnetic anomalies of two dimensional structures (total, horizontal, or vertical anomalies) to their complex gradients (Nabighian, 1972; Green and Stanley, 1975; Rao *et al*., 1981; Mohan *et al*., 1982).

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In seismic studies, HT of the seismic trace was taken to form the complex trace and then starting off with the complex trace, reflection force and immediate phases were obtained in time environment. Thus the geologic structure was reached (Taner *et al.*, 1979).

On the other hand, in the gravity method, the complex gradients of only the vertical fault anomaly are obtained with HT, and then the unknown parameters of the structure have been found (Pinar, 1985). In the self-potential method, Pinar and Akcig (1991) have determined the structure parameters of a sphere.

Up to the present times, the HT has been used extensively only in magnetic and seismic studies. But in the aforementioned studies HT has been used mostly as Fourier Transform (FT). Taner (1979), in this study, obtained HT through convolution by using normalized Hilbert time-domain operator truncated to 19 points.

In this study, the model parameters of which were unsolved so far, self-potential (SP) methods were determined with HT using convolution and FT methods and the results were compared.

## THEORY

The Hilbert Transform (HT) is a mathematical transform function which shifts the phase of a signal as much as π/2 without changing its amplitude. With this definition HT is a linear system, which transforms odd and even functions, with equal amplitude to each other in space or frequency field. Since HT is a linear set, the system should have an input signal, a transfer function and an output function. When a f(r) function, as an input signal, is passed through HT filter, of which the transfer function is given with equation (1), a $f_1(r)$ function, as output signal, is obtained.

$$H(jw) = -j.\text{sgn}(w) \tag{1}$$

$$\text{sgn}(w) = \begin{vmatrix} 1, & w > 0 \\ 0, & w = 0 \\ -1, & w < 0 \end{vmatrix} \tag{2}$$

The following equations are obtained when the frequency spectra of f(r) and $f_1(r)$ are F(w) and $F_1(w)$ respectively:

$$\begin{aligned} &F_1(w) = H(jw).F(w)\text{, and} \\ &F_1(w) = -jF(w)...for....w \geq 0 \\ &F_1(w) = jF(w).....for.....w < 0. \end{aligned} \tag{3}$$

In space domain, the following relationship exists between f(r) and $f_1(r)$

$$g(r) = f(r) + jf_1(r), \tag{4}$$

$r = x + jz$ .

From equations (4), the input signal f(r) and the output signal $f_1(r)$ of HT form a complex function in space domain. The complex function g(r), which is obtained from equation (4), is defined as *ANALYTIC FUNCTION***.** The impulse response h (t) of this filter is

$$h(x) = F^{-1}\left(-j\,\text{sgn}(w)\right) \tag{5}$$

$$h(x) = \frac{1}{\pi x}. \tag{6}$$

The normalised Hilbert space-domain operator is odd, vanishes for even x, and decreases monotonically in magnitude as x increases for odd x. It is usually applied in a modified truncated version (Rabiner and Gold, 1975)

$$-(\pi x)^{-1} = x^{-1}(1 - ej\pi x). \tag{7}$$

From equation (4), the HT between f (x) and $f_1(x)$ in space-domain is defined with the convolution integral (Bracewell, 1965)

$$f_1(x) = h(x) * f(x) \tag{8a}$$

$$f_1(x) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(v)}{x-v}dv. \tag{8b}$$

The HT is seen to be a linear operation and corresponds to a filtering in which the amplitudes of the spectral components are unchanged but the phases are advanced by π/2 for positive frequencies and retarded by π/2 for negative frequencies. In the potential field methods of geophysics (gravity, magnetic and self-potential), the potential or vertical or horizontal gradients of any field component form an analytical signal as in equation (4). Only one of the input function f (x) or the output function $f_1(x)$ is obtained following geophysical studies. The other component is found by using the HT. The HT is obtained by using convolution or FT methods. The amplitude and phase functions of the complex function, which is given by equation (4), is

$$A(x) = (\text{f}(\text{x})^2 + \text{f}_1(\text{x})^2)^{1/2} \tag{9a}$$

$$\Phi(x) = \tan^{-1}\left(\frac{f_1(x)}{f(x)}\right) \tag{9b}$$

In the end, more than one equation, which contain the same structural, parameters f (x), $f_1(x)$, amplitude, and phase) are formed. Using the root values obtained following common solution of these formulae, structural parameters are obtained.

## SELF POTENTIAL METHOD

In the SP method, using HT has identified the parameters that belong to the inclined sheet structures. The structural parameters have been found through the shared solutions of the equations reached as a result of the position of polarization angle of the vertical line. The practice of HT has been realized by using the methods called convolution and FT separately.

### The Inclined Sheet

The V(x) anomaly that is formed by an inclined sheet extending between -∞ and +∞ in the direction y is presented in Fig. 1 The analytic equation such as the potential that this structure will form can be shown as follows (Rao, *et al*. 1981, 1983)

$$V(x) = MLog_e\left[\frac{x^2 + h^2}{(x-a)^2 + H^2}\right] \tag{10}$$

V(x) = potential
h = depth of sheet's upper end
H = h.n = depth of sheet's lower end

$$a = \left[\frac{h(n-1)}{\tan(\alpha)}\right]$$

α = Inclination angle
l = length of sheet

$$M = \left(\frac{qI}{2\pi}\right)$$

d = distance between the start of profile and the projection of sheet's upper end on surface
X = x-d.

The complex gradients of V(x) found through the equation in ($T_x$ in the direction of x, $T_z$ in the direction of z) form the following equations (Fig. 2)

$$V(x) = \frac{\partial V(x)}{\partial x} + i\frac{\partial V(x)}{\partial z} \tag{11}$$

$$T_x = \frac{\partial V(x)}{\partial x} = 2M\left[\frac{x}{X^2 + h^2} - \frac{(X-a)}{(X-a)^2 + n^2h^2}\right] \tag{12}$$

$$T_z = \frac{\partial V(x)}{\partial z} = 2M\left[\frac{h}{X^2 + h^2} - \frac{(X-a)}{(X-a)^2 + n^2h^2}\right] \tag{13}$$

In the SP method the following relation is also available between the horizontal and vertical gradients

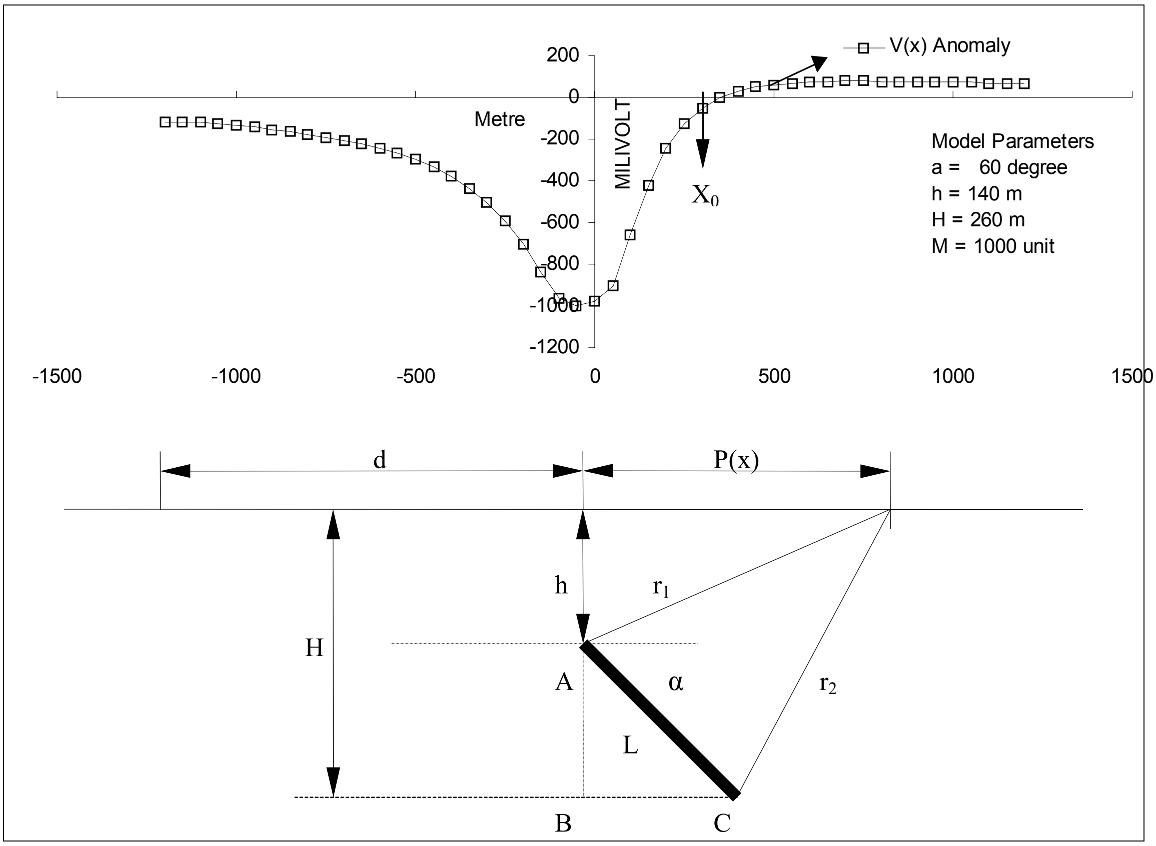


**FIG. 1.** Cross-sectional view of 2-D thin inclined sheet model

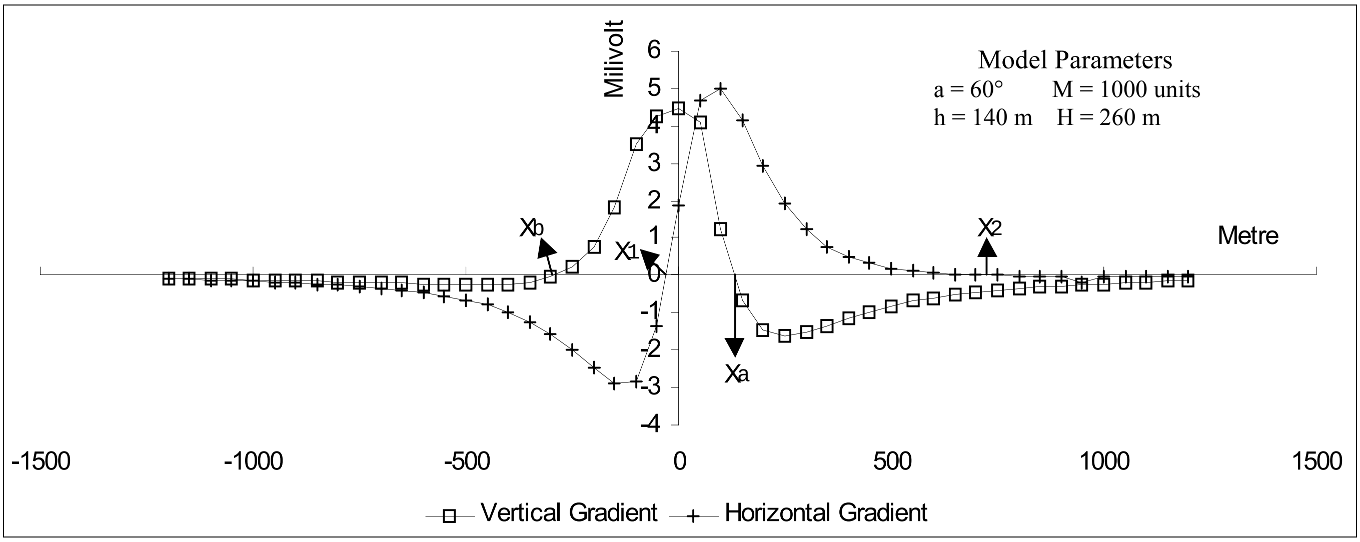


**FIG. 2.** Complex gradient functions of V(x) belonging to thin inclined sheet model.

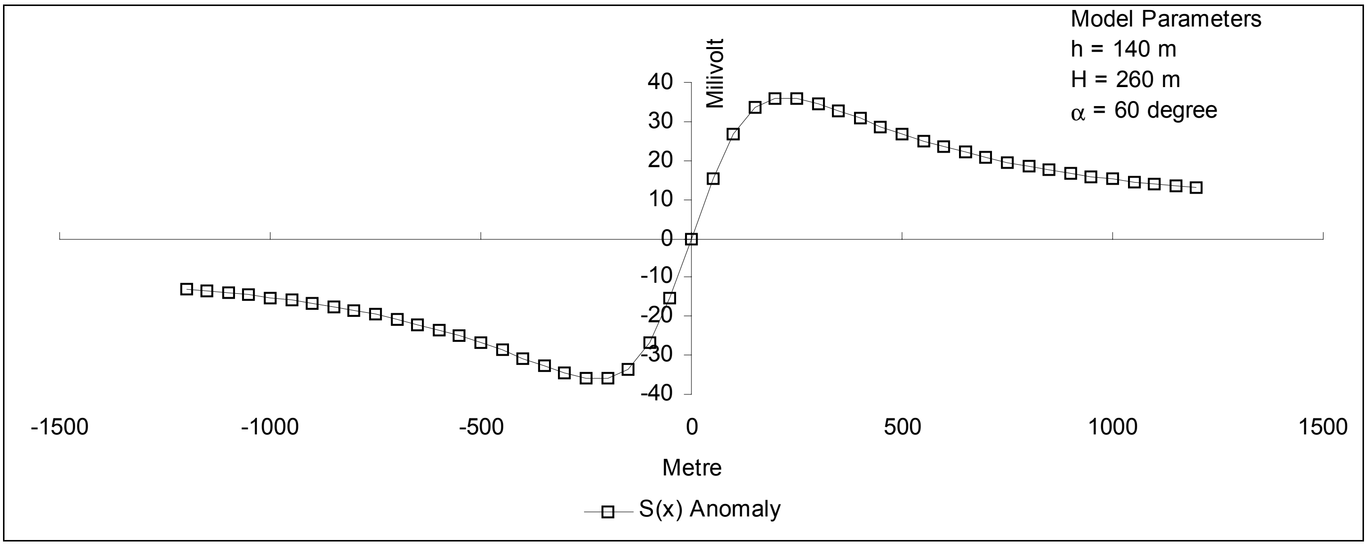


**FIG. 3.** S(x) anomaly of thin inclined sheet model.

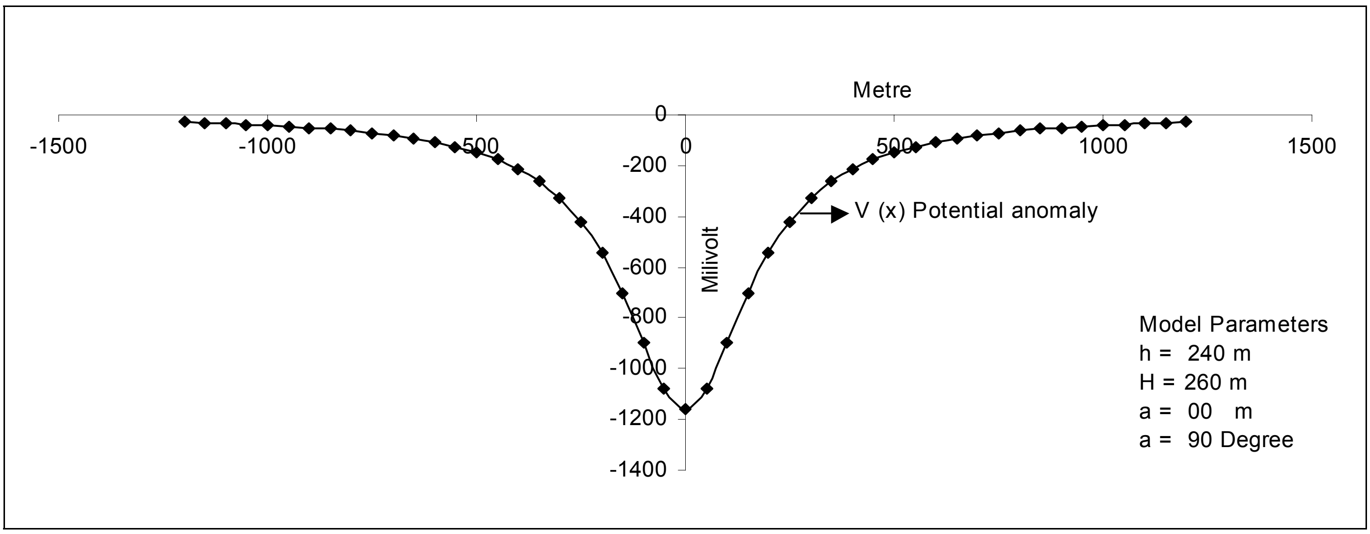


**FIG. 4.** V(x) anomaly of thin vertical sheet model.

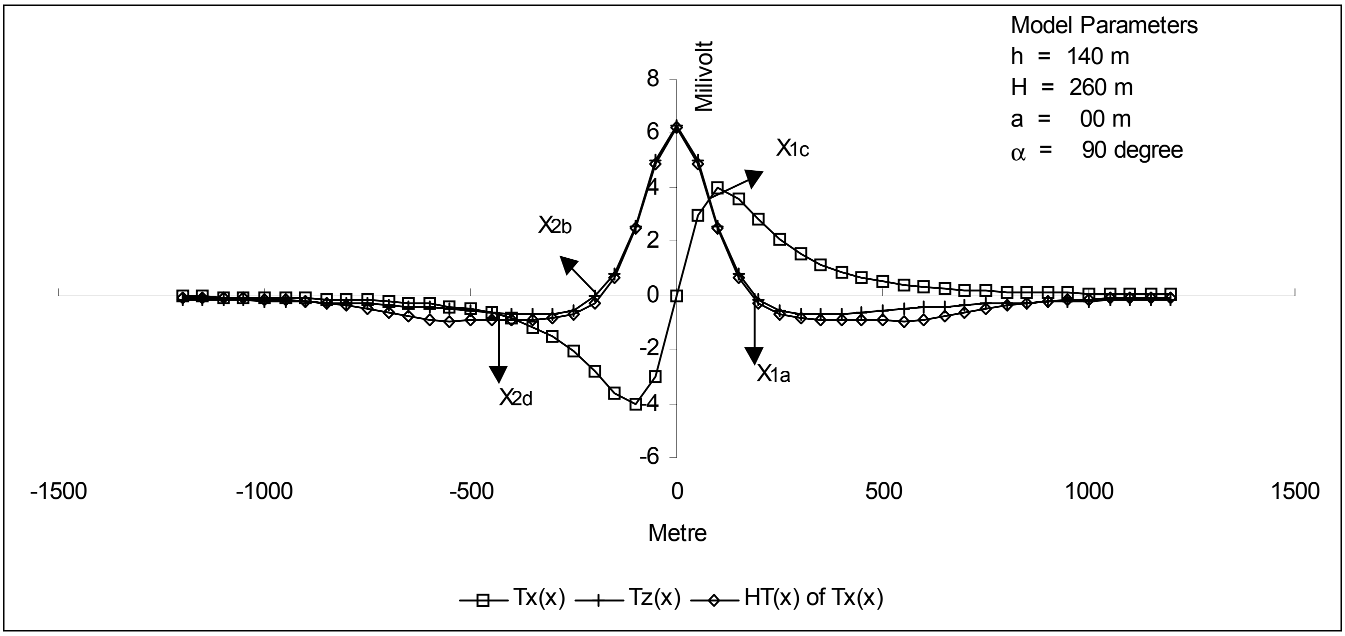


**FIG. 5.** Complex gradients functions of V(x) belonging to thin vertical sheet model.

$$T_x(x) \xleftrightarrow{HT} -T_z(x) \tag{14}$$

Besides, making use of the equations and the amplitude and phase we find function

$$A(x) = \left[T_x(x)^2 + T_z(x)^2\right]^{1/2} \tag{15}$$

$$\Phi(x) = \tan^{-1}\left(\frac{T_z(x)}{T_x(x)}\right) \tag{16}$$

The unknown parameters of the structure in the equation (10) are α, n, h, H, l, M and x=0. We use the shared solutions of the equations in (10), (12) and (13) in order to find these parameters by means of HT. For these solutions;

**a)** The root is found through the equation (10) under the condition of V(x) = 0 (Fig 2)

$$X_0 = \frac{a^2 + h^2(n^2 - 1)}{2a} \tag{17}$$

**b)** For $T_x(x) = 0$ using the equation (12), the following equations are obtained:

$$X^2 - \frac{X\left[a^2 + h^2(n^2 - 1)\right]}{a} - h^2 = 0 \tag{18}$$

When the result of (17) is replaced equation (18), the following equation is reached:

$$X^2 - 2X_0X - h^2 = 0 \tag{19}$$

and from the solution of equation (19), the following two roots are obtained (Fig. 2):

$$X_1 = X_0 + (X_0{}^2 + h^2)^{1/2} \quad \text{(Maximum root)} \tag{20}$$

$$X_2 = X_0 - (X_0{}^2 + h^2)^{1/2} \quad \text{(Minimum root)} \tag{21}$$

**c)** From $T_z(x) = 0$ solution (through the equation (13), the following equation

$$X^2(1-n) - 2aX + (a^2 + h^2(n^2 - n)) = 0, \tag{22}$$

and the following two roots:

$$X_a = \frac{a + \left\{n\left[a^2 + h^2(n-1)\right]\right\}^{1/2}}{(1-n)} \tag{23}$$

$$X_b = \frac{a - \left\{n\left[a^2 + h^2(n-1)\right]\right\}^{1/2}}{(1-n)} \tag{24}$$

are determined. The body parameters (x=0 point**,** h, H, α and L) are found in the following way by means of the roots found in **a, b** and **c**

**d)** If equation (10) is reorganized by using x=0 point antisymetric functions and by considering the following approaches,

$$S(X) = -S(-X)$$

$$S(X) = \frac{\left[V(x) - V(-X)\right]}{2} \tag{25}$$

The following equation is obtained (Rao, *et al.*, and 1983):

$$S(X) = MLog_e\left[\frac{(X+a)^2 + H^2}{(X-a)^2 + H^2}\right] \tag{26}$$

The condition of S(0)=0 is used in equation (25). As for its definition, S(x) function must always have the value of zero in the condition of X=0 (Fig. 3).

**e)** We obtain the h parameter from the shared solutions of equations (20) and (21) (Rao, *et al*., 1983)

$$h=\left[\left|X_1 X_2\right|\right] \tag{27}$$

The square root of the absolute value obtained by multiplication of the roots of $T_x(x)$ inclined sheet gives the top extent depth of the inclined sheet.

**f)** In order to find the H parameter, first of all, the n value in the equation of H = nh must be found. To this end, using the equation (17), the following equation is obtained.

$$a=X_0-\left[X_0^{\,2}-h^2\left(n^2-1\right)\right]^{1/2}. \tag{28}$$

The (23) and (24) total of the roots that belong to $T_z(x)$ anomaly is formed

$$\left(X_a+X_b\right)=\frac{2a}{(1-n)} \tag{29}$$

and when the equation in (28) is replaced in equation (29), the following equation is reached:

$$n=\left[\frac{\left(X_a+X_b\right)^2-4X_0\left(X_a+X_b\right)+4X_1X_2}{\left(X_a+X_b\right)^2-4X_1X_2}\right] \tag{30}$$

and when the (n) parameter is used in the equation H=n.h depth is obtained.

**g)** As the other parameters of the sheet (□ and L) are related to each other through parameter (a), first of all, the value (a) must be found. To do this, equation (30) is replaced in (29) and the following equation is obtained:

$$a=\left[\frac{2X_0\left(X_a+X_b\right)^2-4X_1X_2\left(X_a+X_b\right)}{\left(X_a+X_b\right)^2-4X_1X_2}\right] \tag{31}$$

In this equation, the previously obtained roots are replaced and parameter (**a**) can be determined.

**h)** The inclination of the sheet α and the M constant are, however, obtained by the use of h (18), n (30) and a (31) altogether.

$$\alpha=\left[\tan^{-1}\frac{h(n-1)}{a}\right] \tag{32}$$

$$M=\frac{\left[T_x(x=0)\left(a^2+h^2n^2\right)\right]}{2a} \tag{33}$$

**i)** The length of the sheet (L) is, on the other hand, obtained through the equation below by using H, h and □ values in ABC triangle as seen in Fig 2.

$$L=\frac{\left[h(n-1)\right]}{\sin(\alpha)} \tag{34}$$

The results obtained from the various theoretical models are given Table 1 and Fig. 8.

## The Vertical Sheet

These solutions are true for α ≠ 90°. However in the condition of α = 90°, the structure parameters are found by means of the equations below. For α = 90°, the equation in (10) transforms to the following equation (Fig. 4) (Roa, *et al*., 1983)

**Table 1**. Application of nomogram and theoretical HT in SP method for inclined and vertical sheet models.

| Parameters | | h [m] | H [m] | L [m] | α [°] | a [m] | d [m] |
|---|---|---|---|---|---|---|---|
| Model 1 | Assumed values | 140 | 220 | 105 | 45 | 74 | 1200 |
| | HT with convolution | 143 | 199 | 75 | 49 | 49 | 1200 |
| | Nomogram (Murty and Haricharan, 1985) | 142 | 198 | 85 | 50 | 50 | 1200 |
| Model 2 | Assumed values | 140 | 220 | 84 | 60 | 40 | 1200 |
| | HT with convolution | 146 | 200 | 64 | 63 | 27 | 1200 |
| | Nomogram (Murty and Haricharan, 1985) | 145 | 200 | 74 | 65 | 30 | 1200 |
| Model 3 | Assumed values | 140 | 260 | 120 | 90 | 00 | 1200 |
| | HT with convolution | 162 | 212 | 172 | 90 | 00 | 1200 |
| | Power Spektrum | 162 | 215 | 175 | 90 | 00 | 1200 |
| Model 4 | Assumed values | 140 | 220 | 106 | 135 | 74 | 1200 |
| | HT with convolution | 143 | 200 | 75 | 131 | 49 | 1200 |
| | Nomogram (Murty and Haricharan, 1985) | 145 | 200 | 85 | 130 | 50 | 1200 |

**Table 2**. Application of nomogram and theoretical HT in SP method for infinite horizontal circular cylinder.

| Parameters | | h [m] | M | α [°] | d [m] |
|---|---|---|---|---|---|
| Model 1 | Assumed Values | 60 | 1000 | 45 | 400 |
| | HT with convolution | 60 | 1000 | 45 | 400 |
| | Nomogram (Bhattacharya and Roy, 1982) | 60 | 1000 | 45 | 400 |
| | Power Spectra | 60 | | | |
| Model 2 | Assumed Values | 80 | 1000 | 60 | 400 |
| | HT with convolution | 80 | 1000 | 60 | 400 |
| | Nomogram (Bhattacharya and Roy, 1981) | 80 | 1000 | 60 | 400 |
| | Power Spectrum | 80 | | | |
| Model 3 | Assumed Values | 60 | 1000 | 135 | 400 |
| | HT with convolution | 60 | 1000 | 135 | 400 |
| | Nomogram (Bhattacharya and Roy, 1981) | 60 | 1000 | 135 | 400 |
| | Power Spectra | 60 | | | |

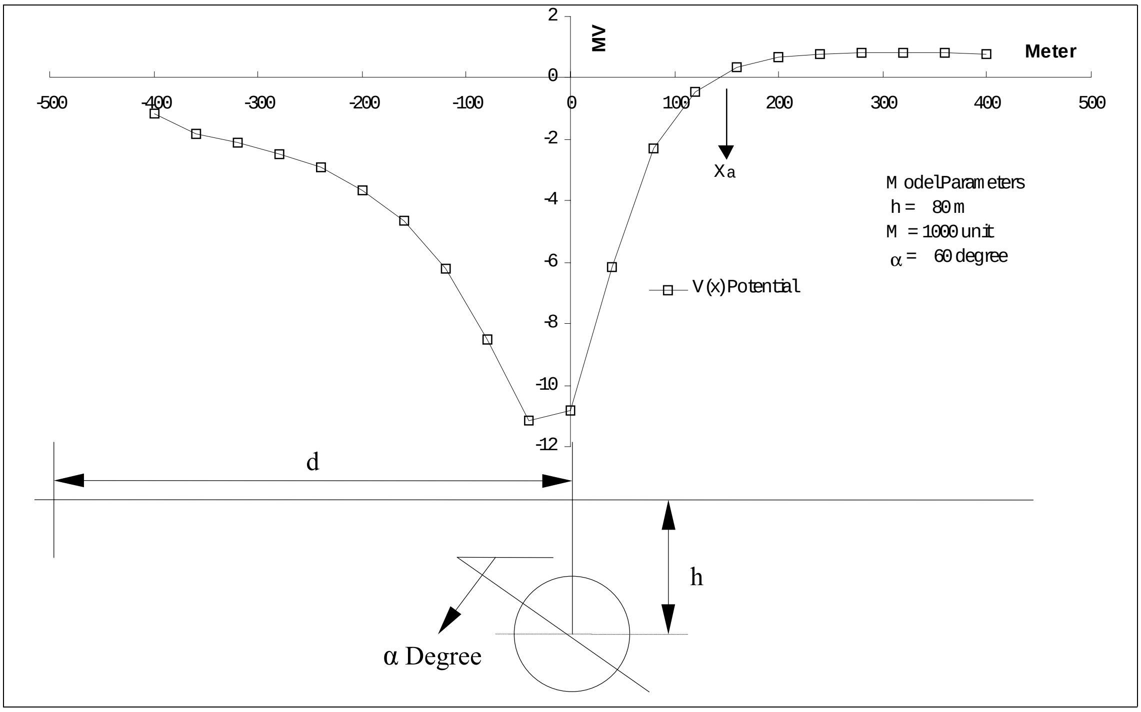


**FIG. 6.** Cross-sectional view and V(x) anomaly of 2-D cylinder model

**Table 3**. Application of HT in SP profiles belonging to Semenov 1968 using inclined sheet model.

| Parameters | | h [m] | H [m] | α [°] | M | d [m] |
|---|---|---|---|---|---|---|
| Profile | HT using Convolution | 25 | 64 | 75 | 210 | 325 |
| | Nomogram (Murty and Haricharan, 1985) | 20 | 60 | 70 | 200 | 320 |

**Table 4**. Application of HT in SP profiles belonging to Etibank using infinite horizontal circular cylinder.

| Parameters | | h [m] | α [°] | M | D [m] |
|---|---|---|---|---|---|
| Profile | HT using Convolution | 15 | 86 | 420 | 100 |
| | Nomogram (Bhattacharya and Roy, 1981) | 15 | 90 | 400 | 100 |
| | Power Spectra | 15 | | | |

$$V(x) = M \log_e \left[ \frac{X^2 + h^2}{X^2 + H^2} \right] \quad (35)$$

V(x) complex gradients are found as $T_x(x)$ and $T_z(x)$ as follows (Fig. 5):

$$T_X(X) = \frac{\partial V(X)}{\partial X} = 2M \left[ \frac{X}{X^2 + h^2} - \frac{X}{X^2 + H^2} \right] \quad (36)$$

$$T_z(X) = \frac{\partial V(X)}{\partial z} = 2M \left[ \frac{h}{X^2 + h^2} - \frac{H}{X^2 + H^2} \right] \quad (37)$$

The mathematical solutions of the equations in (35), (36) and (37) are done in order to find the structural parameters (d, h, H, L and M).

**a)** For (d) parameter,
V (x=0) = max or min point

$$T_x\ (x=0) = 0 \quad (38)$$

$T_Z$(x=0) max or min point

**b)** For $T_z(x) = 0$, the following roots are found (Fig.5)

$$X_{1a} = (hH)^{1/2} \quad (39)$$

$$X_{2b} = -(hH)^{1/2} \quad (40)$$

**c)** From the equation of $T_x(x) = T_z(x)$, we obtain the following equations

$$X^2(H-h) + X\left(H^2 - h^2\right) + \left(h^2 H - hH^2\right) = 0$$

and from the solution of this equation, the following roots are obtained (Fig.5)

$$X_{1c} = \frac{-(2h+L)}{2} + \frac{\left(L^2 + 8X_{1a}^{\ 2}\right)^{1/2}}{2} \quad (41)$$

$$X_{2d} = \frac{-(2h+L)}{2} - \frac{\left(L^2 + 8X_{1a}^{\ 2}\right)^{1/2}}{2} \quad (42)$$

The (L) parameter is calculated using the equation below by means of the shared solutions of the roots.

$$L = \left[\left|(X_{1c} - X_{2d})^2 - 8X_{1a}^{\ 2}\right|\right]^{1/2} \quad (43)$$

**d)** As for $T_z(x=0)$, the following equation is obtained;

$$T_z(X=0) = 2M\left(\frac{1}{h} - \frac{1}{H}\right) \quad (44)$$

From the simultaneous solutions of (39) and (44), parameter (M) is found as follows:

$$M = \frac{\left[Tz(X=0) X_{1a}^{\ 2}\right]}{2L} \quad (45)$$

**e)** As for V(x=0), the equation in (35) becomes

$$V(X=0) = M\, Log_e\left(\frac{h^2}{H^2}\right) \quad (46)$$

If the logarithms of the two sides of the equation above are calculated, the following equations are obtained:

$$Log_e V(X=0) = Log_e M + \frac{h^2}{H^2}$$

$$Log_e\left[\frac{V(X=0)}{M} = \frac{h^2}{H^2}\right] \quad (47)$$

The following equations are reached through the equation (39):

$$H = \frac{X_{1a}^{\ 2}}{h} \quad (48)$$

The simultaneous solution of the equation (47) and (48) gives the parameter (h)

$$h = \left[ X_{1a}^{\ 2} \left\{ e^{\frac{V(X=0)}{M}} \right\}^{1/2} \right]^{1/2} \quad (49)$$

**f)** The equation (49) is replaced in (48) to find (H) parameter; it is defined by equation below:

$$H = \frac{X_{1a}^{\ 2}}{\left[ X_{1a}^{\ 2} \left\{ e^{\frac{V(X=0)}{M}} \right\}^{1/2} \right]^{1/2}} \quad (50)$$

Application of nomogram and theoretical HT for various theoretical inclined sheet models are given Table 1.

## Horizontal Cylinder

The V(x) anomaly that is formed by a horizontal cylinder (Fig. 6) whose inclination angle is α is given below (Bhattacharryya and Roy, 1981)

$$V(x) = M \frac{(x-d)\cos(\alpha) - h\sin(\alpha)}{(x-d)^2 + h^2} \quad (51)$$

The relations modified:
V(x) = potential
α = inclination angle
h = the cylinder focal depth
M = coefficient

**Table 5**. Application of nomogram and HT in SP profiles belonging to Kuºadasi-Aydin using inclined sheet model.

| Profiles | Parameters | h [m] | H [m] | α [°] | M | d [m] |
|---|---|---|---|---|---|---|
| A – A' | HT using Convolution | 25 | 64 | 75 | 210 | 325 |
| | Nomogram (Murty and Haricharan, 1985) | 20 | 60 | 70 | 200 | 320 |
| B – B' | HT using Convolution | 60 | 180 | 57 | 170 | 275 |
| | Nomogram (Murty and Haricharan, 1985) | 55 | 175 | 60 | 160 | 270 |

**Table 6**. Application of nomogram and HT in SP profiles belonging to Kuºadasi-Aydin using horizontal cylinder model

| Parameters | | h [m] | α [°] | M | D [m] |
|---|---|---|---|---|---|
| A - A' Profile | HT using Convolution | 15 | 86 | 420 | 325 |
| | Nomogram (Bhattacharya and Roy, 1981) | 15 | 80 | 400 | 320 |
| | Power Spectra | | | | |
| B – B' Profile | HT using Convolution | 37 | 73 | 690 | 275 |
| | Nomogram (Bhattacharya and Roy, 1981) | 35 | 70 | 6500 | 270 |
| | Power spectra | 35 | | | |

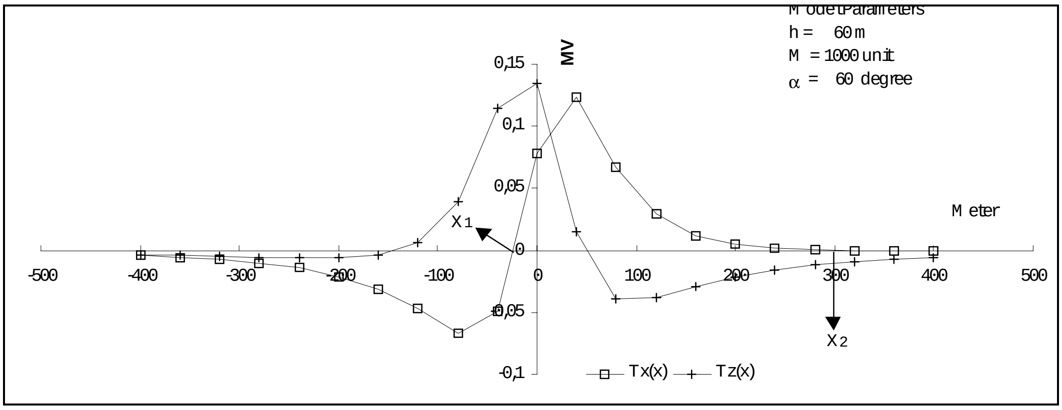


**FIG. 7.** Complex gradients functions of V(x) belonging to cylinder model.

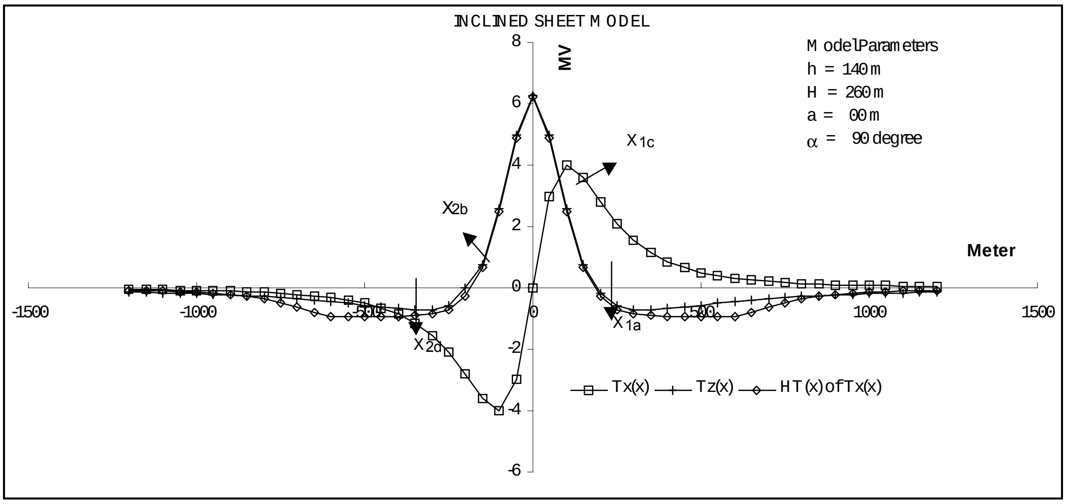


**FIG. 8.** Hilbert Transform applications of $T_x(x)$ using convolution operator.

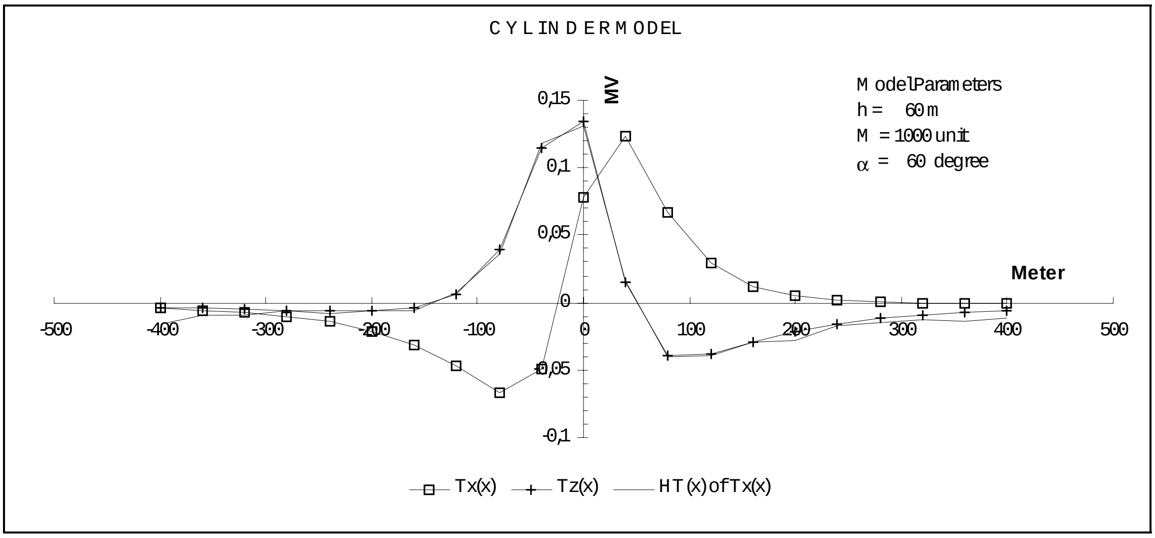


**FIG. 9.** Hilbert Transform applications of $T_x(x)$ using convolution operator.

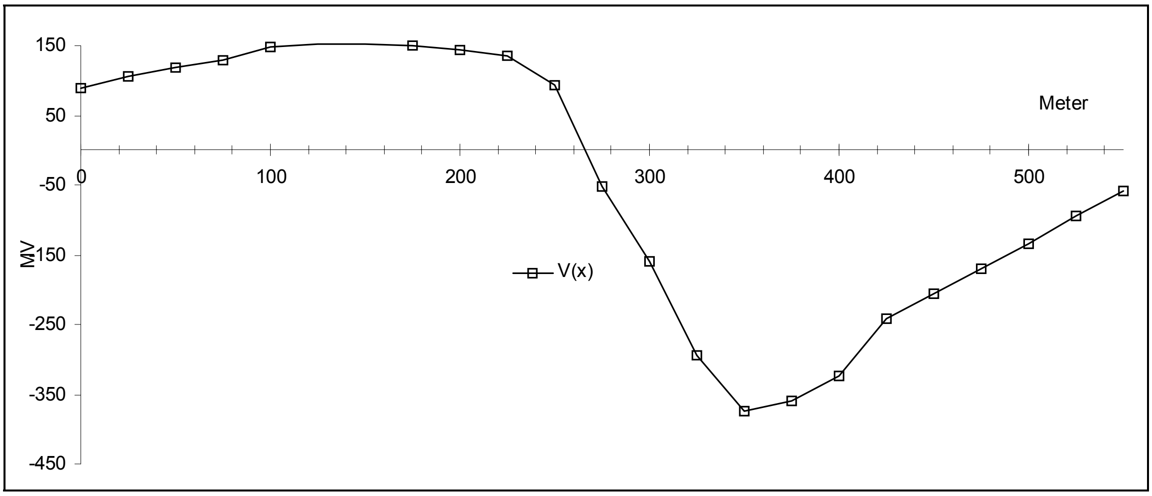


**FIG. 10.** Self potential V(x) anomaly (from Semenov 1985)

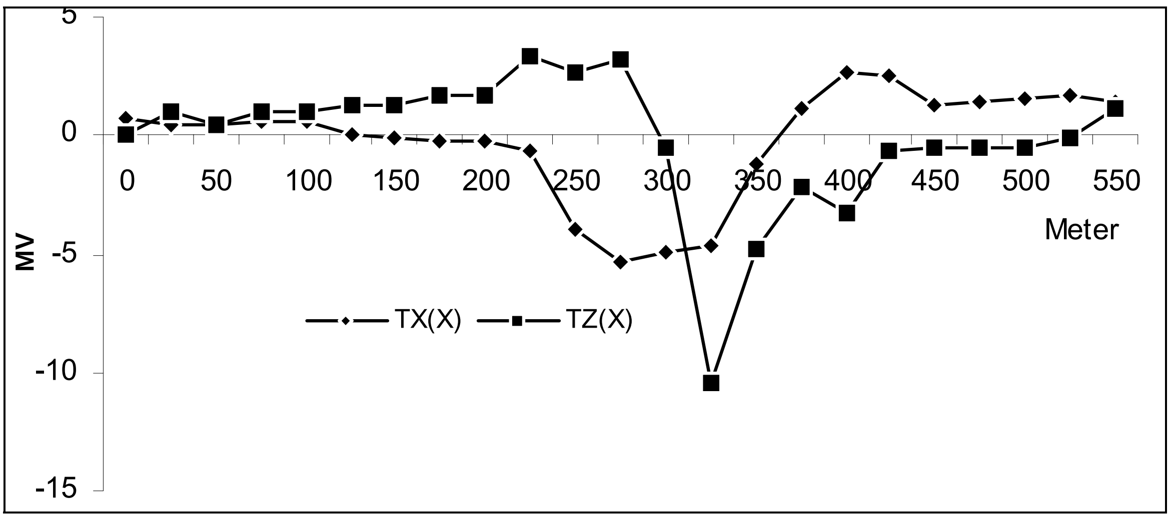


**FIG. 11.** Complex gradients of V(x) (from Semenov 1985)

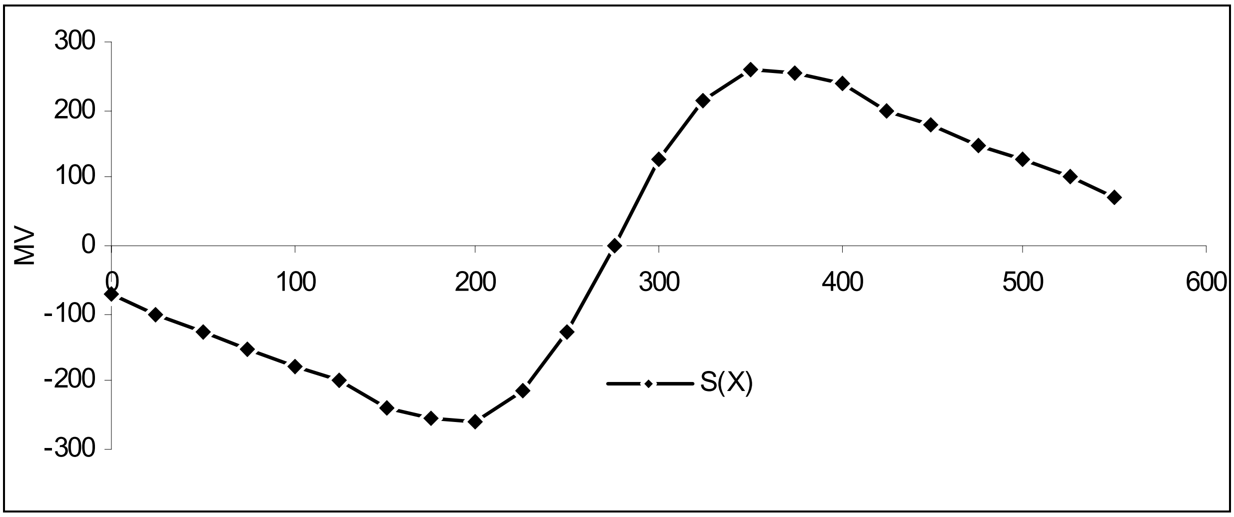


**FIG 12.** S(x) function of V(x) (from Semenov 1985)

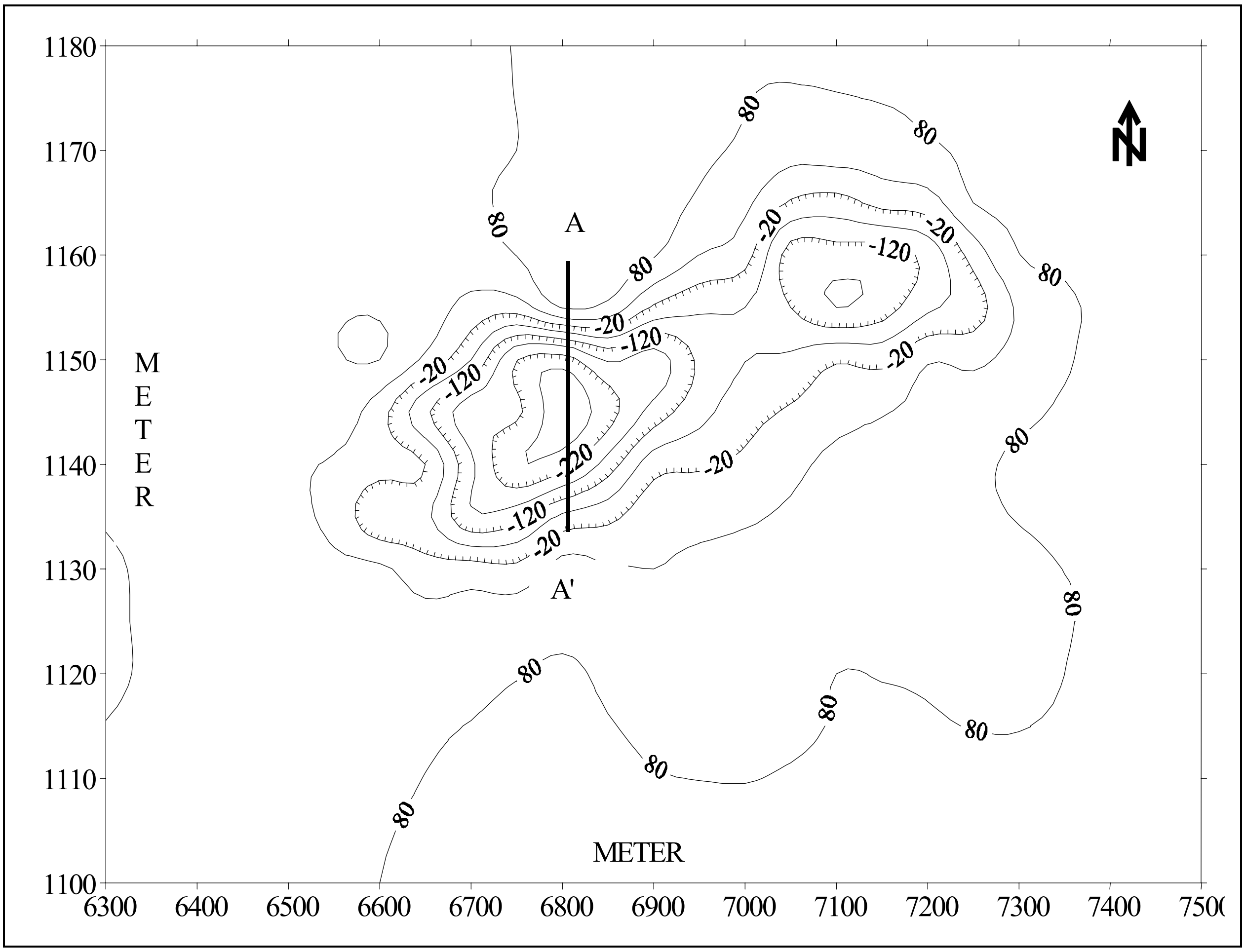


**FIG. 13.** Anomaly map of V(x) (from Etibank 1957)

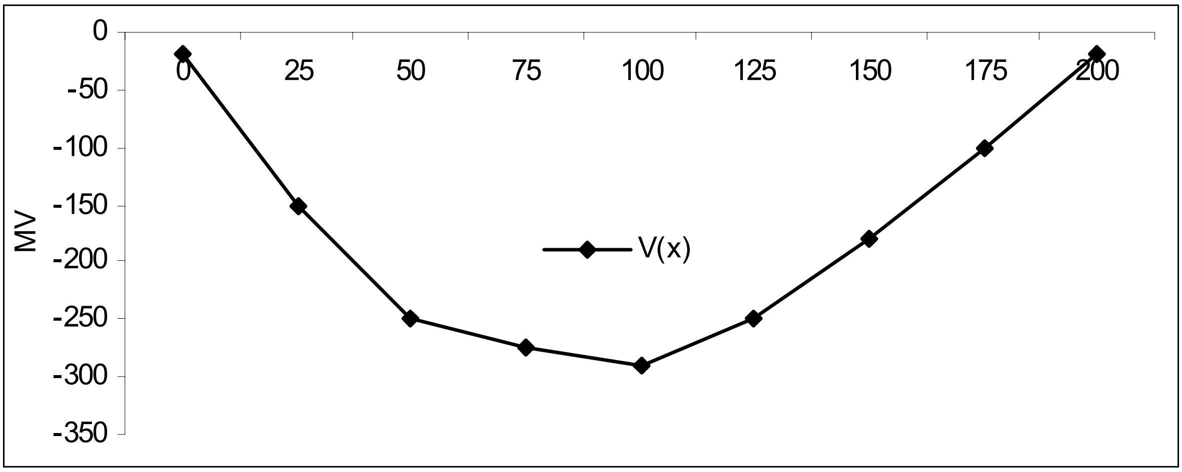


**FIG. 14.** A-A' profile from self potential V(x) anomaly map (from Etibank 1957)

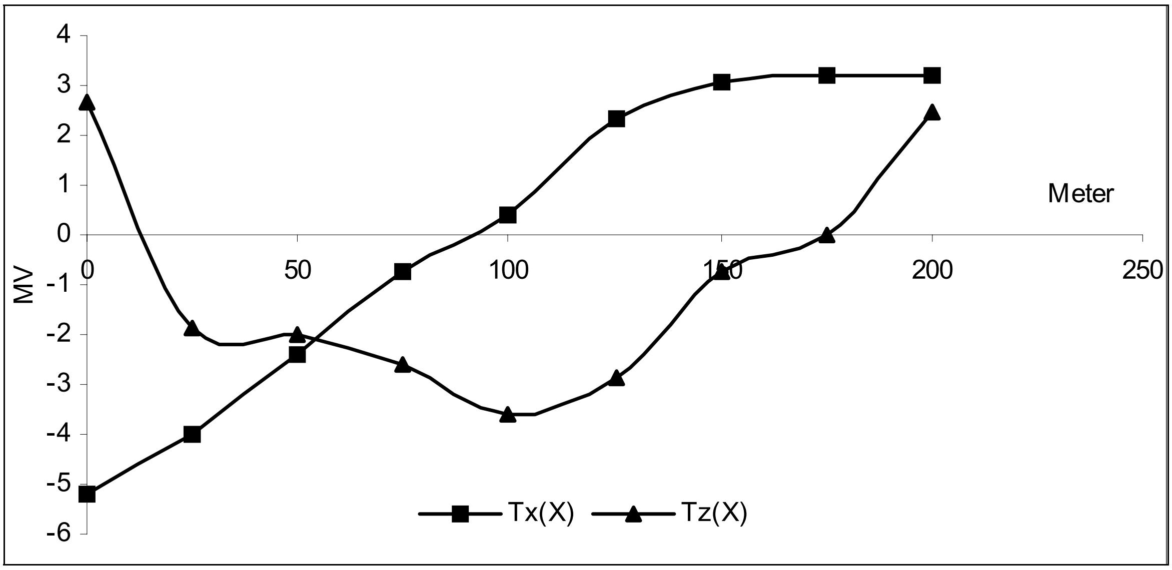


**FIG. 15.** Complex gradients of A-A' profile (from Etibank 1957)

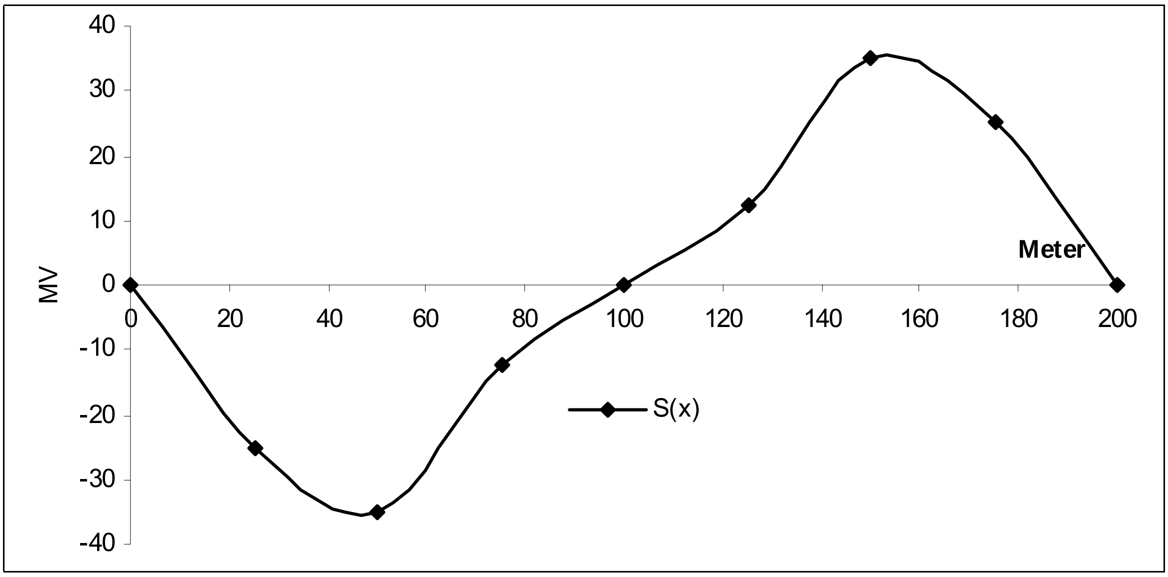


**FIG. 16.** S(x) function of V(x) (Murgul-Etibank 1957)

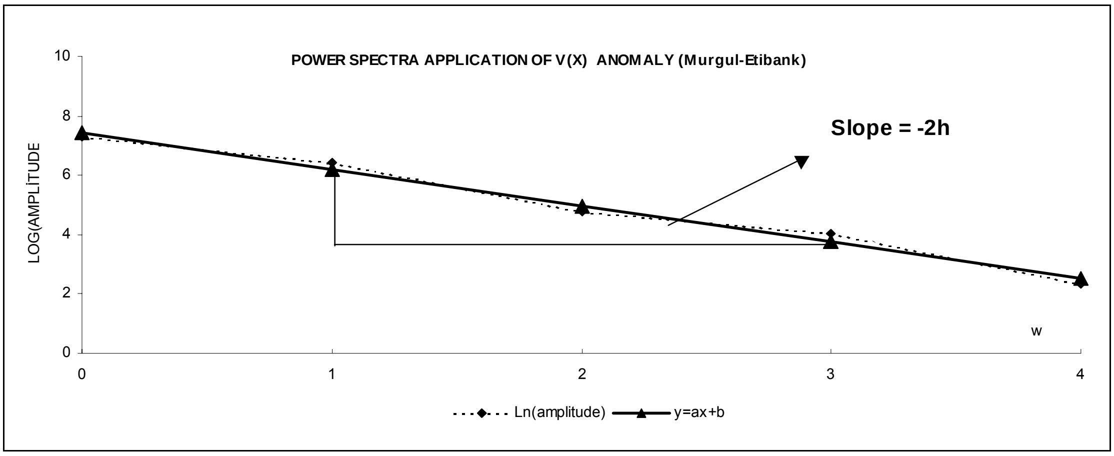


**FIG. 17.** Power spectra function of A-A' profile

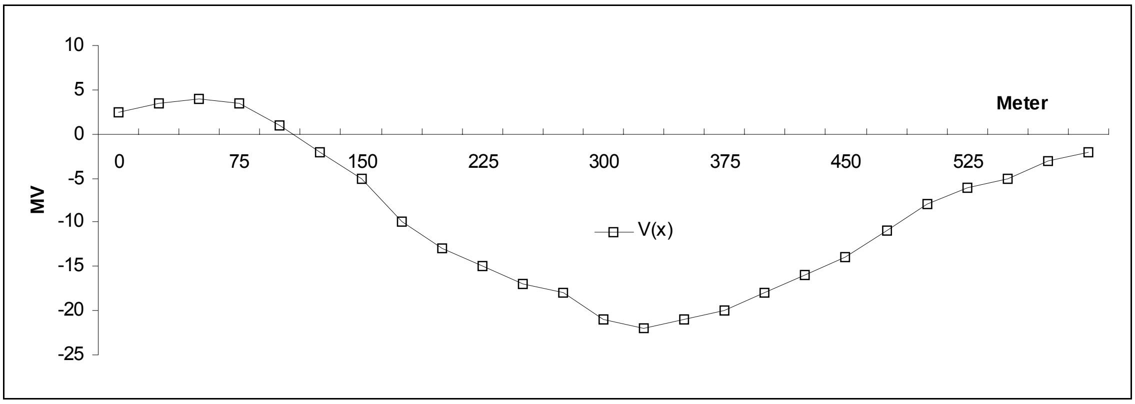


**FIG. 18.** A-A' profile (from Kuşadasi - Aydin)

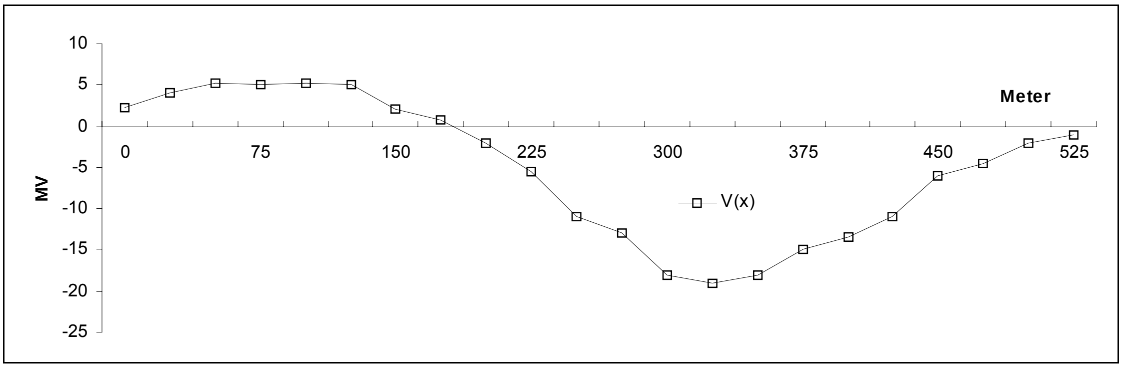


**FIG. 19.** B-B' profile (from Kuşadasi - Aydin)

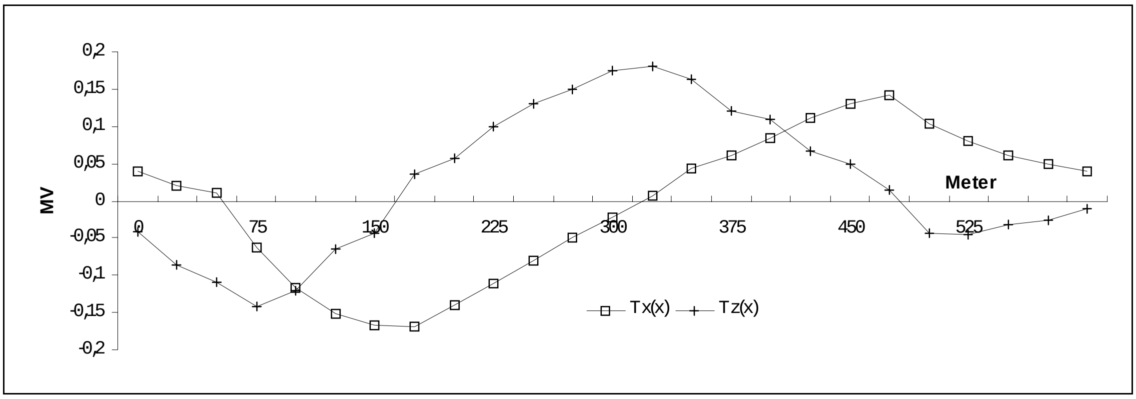


**FIG. 20.** Complex gradients of A-A' profile (from Kuşadasi - Aydin)

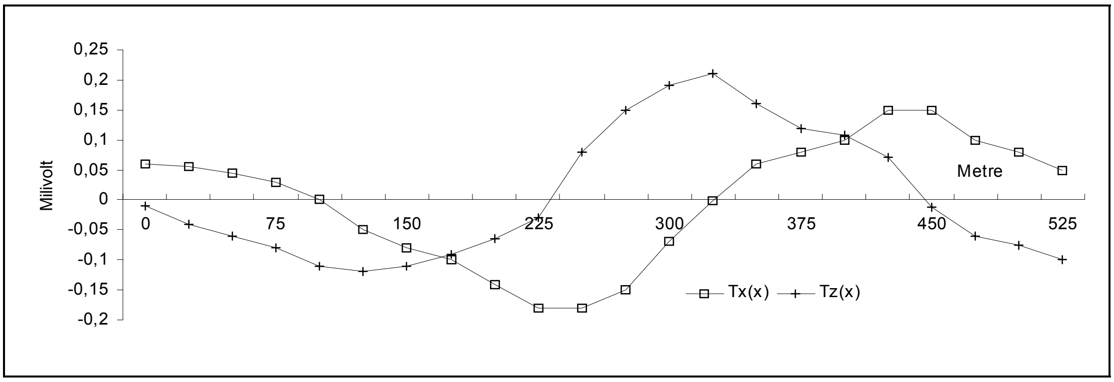


**FIG. 21.** Complex gradients of B-B' profile (from Kuşadasi - Aydin)

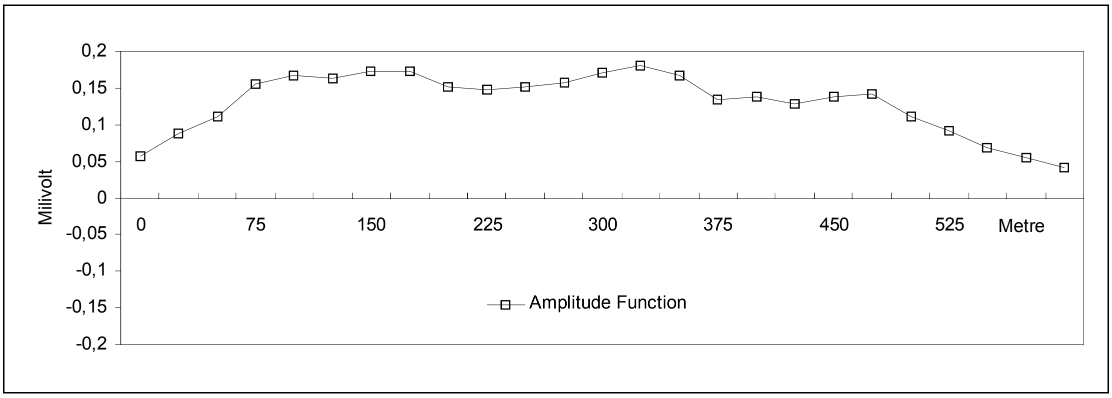


**FIG. 22.** Amplitude function of A-A' profile (from Kuşadasi - Aydin)

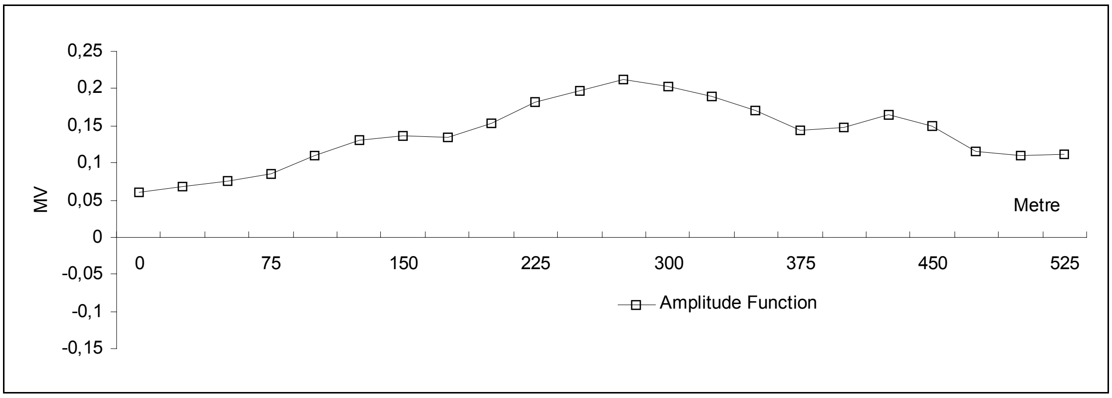


**FIG. 23.** Amplitude function of B-B' profile (from Kuşadasi - Aydin)

d = distance between profile beginning and the projection of cylinder's center on surface
X = (x-d).

From the equation (11) and (51), the complex gradients of V(x) are given the following equations (Fig. 7).

$$Tx(x) = M\left[\frac{\cos(\alpha)\left(h^2 - X^2\right) + 2\,X\,h\sin(\alpha)}{\left(X^2 + h^2\right)^2}\right] \qquad (52)$$

$$Tz(x) = M\left[\frac{\sin(\alpha)\left(h^2 - X^2\right) - 2\,X\,h\,\cos(\alpha)}{\left(X^2 + h^2\right)^2}\right] \qquad (53)$$

Additionally, the amplitude and phase functions are obtained by using the equations (15) and (16). The unknown parameters of the structure in the equation (51) are α, h, m and d. We use the shared solutions of the equations (51), (52), (53), (15) and (16) in order to find these parameters by means of HT. For these solutions;

**a)** For V(x) = 0, using the equation (51), the following equation is obtained

$$XcCos(\alpha) - h\ \sin(\alpha) = 0.$$

And from the solutions of the below equation, the following root is obtained (Fig. 7 ):

$$X_a = h\ \tan(\alpha) \tag{54}$$

**b)** From $T_x(X)=0$ solution, through the equation (52), the following equation

$$-X^2\cos(\alpha) + 2\,X\,h\,\sin(\alpha) + h^2\cos(\alpha) = 0$$

and the following two roots

$$X_1 = \frac{h\sin(\alpha) + h}{\cos(\alpha)} \tag{55}$$

$$X_2 = \frac{h\sin(\alpha) - h}{\cos(\alpha)} \tag{56}$$

are found (Fig. 7).

**c)** From the solutions of the equations (52) and (53), we obtained the following equations

$$T_X(X=0) = M\left[\frac{\cos(\alpha)}{h^2}\right] = c \tag{57}$$

$$T_Z(X=0) = M\left[\frac{\sin(\alpha)}{h^2}\right] = d \tag{58}$$

$$\tan(\alpha) = \frac{d}{c} \tag{59}$$

The body parameters α, h, m and d are found in the following way by means of the roots and equations in a, b and c.

**d)** In order to find the parameter d (X=0), the property which is obtained through the relation (15) is used (Fig. 6)

$$A(X=0) = \text{maximum value} \tag{60}$$

**e)** The parameters α and h are obtained by using the roots in $X_a$, $X_1$, $X_2$ and from equations (57), (58) and (59). By employing the results in (55) and (56), we get

$$X_1 - X_2 = c\frac{2h}{\cos(\alpha)} = X_m \tag{61}$$

The parameters α and h can be found from both the equation (59)and (61). If the equation (61) is used;

1) The parameter α is obtained through the relations (54) and (59) as

$$\alpha = \sin^{-1}\left(\frac{2X_a}{X_m}\right) \tag{62}$$

2) If the equation in (62) is put its place in (54) to find **h** parameter, it is defined by equation below

$$h = t\frac{Xa}{\tan(\alpha)} \tag{63}$$

If the equation (59) is used, the parameters α and h are obtained through the relations (56) and (59) as

$$\alpha = \tan^{-1}\left(\frac{d}{c}\right) \tag{64}$$

$$h = \frac{X_a}{\tan(\alpha)} \tag{65}$$

3) From the shared solutions of (51), (62), (63), (64) and (65), parameter M is found as follows

$$M = \frac{V(X=0)h}{\sin(\alpha)} \tag{66}$$

Application of nomogram, power spectrum and theoretical HT for various theoretical horizontal cylinder models are given Fig. 9 and Table 2.

## FIELD EXAMPLES

The anomaly from Seminov (1985) (Çağlar, 1991) was used in order to examine HT method in deeping sheet model (Fig.10). Firstly, the nomogram method (Murty *et al.*, 1985) was used, then the evaluations were performed by using HT method (Fig. 10, 11 and 12) and the results from these two methods were compared (Table 3).

Spontaneous potential data of the Murgul Copper works, which were collected by Etibank in 1957, were used for the horizontal cylinder model (Figs. 13 and 14). Profile AA' was assumed to be horizontal cylinder. It was analysed with nomogram and power spectra methods (Akgün *et al.*, 1996) and then the HT solutions were carried out on them (Figs. 15, 16 and 17; Table 4).

The field of Davutlar-Kuşadasi-Aydin which is a geothermal area at the extension of the Büyük Menderes Graben system alongside of the Agean Sea, were investigated for the purpose of saline water intrusion. From the SP anomalies they could be considered as a cylinder or an inclined sheet models and accordingly the position and extent of saline of saline intrusion levels can be determine. Two SP anomalies (Figs. 18 and 19) were interpreted with HT method using convolution (Figs. 20, 21, 22 and 23). The body parameters were determined according to inclined sheet and horizontal cylindrical models. The results are given in Table 5 and 6.

## CONCLUSION AND SUGGESTIONS

The results of this study are given below.

1. The parameters were determined exactly for the theoretical models using the HT method. Especially, the location of the structure, which had not been determined up to now, was obtained precisely directly from the anomaly for the self-potential method.

2. Before the interpretation of the field data with the HT method, the anomaly should be refined from noise. If this procedure hasn't been carried out, pseudo roots could be formed in the complex gradients of anomaly.

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