JOURNAL OF THE BALKAN GEOPHYSICAL SOCIETY, Vol. 4, No 2, May 2001, p. 45-50, 5 figs.

Integral DMO revisited

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(Received 21 February 2000; accepted 6 November 2000)

***Abstract**: It is extremely important to investigate the variations as to the amplitude and phase of seismic wavelets in relative amplitude processing. This crucial point should be taken into consideration in terms of all the phases of a processing sequence. In this paper, DMO will be studied and analyzed with respect to amplitude and phase. Our objective is to find the amplitude correction term which must be applied to data reflected from dipping reflectors in order to insure amplitude preservation no matter what the value of dip angle is.*

***Key Words**: DMO, NMO, CMP, Constant Offset Gather.*

INTRODUCTION

The major goal of seismic data processing is to obtain the reflectivity function distributed within the earth and keep the relative amplitude relations constant among geological units so that the end product represents the actual physical environment seismic waves pass through. In order to establish this goal, reflected events must be migrated to their corresponding spatial coordinates and the peak amplitudes of migrated events must be proportional to the reflectivity. An in-depth explanation of this issue may be found in the technical study prepared by Black *et al*. (1993).

Relative amplitude basic processing sequence may be considered as the sub-sequent application of spherical-divergence, normal move out (NMO), dip move out (DMO) corrections and zero offset migration as well as the implementation of some optional algorithms correctly in a constant velocity medium. If velocity varies spatially, driving mathematical formulas for DMO and migration is extremely difficult. Therefore, some implementations are made at the expense of something else. The subject of DMO has been studied by many distinguished authors such as Yilmaz and Clearbout (1980), Deregowski and Rocca (1981), Hale (1984). Even tough, each mentioned effort made invaluable contribution to the reflection method they could not handle the amplitude distribution dynamically. Black *et al.* (1993) corrected the subtle flow in the derivation of Hale's approach and ended up with true amplitude and phase spectrums. The flaw observed in Hale's derivation was the ignorance of migration of input event to its zero-offset location spatially (reflection point smear). Since the phase of Hale DMO operator was correct, events are repositioned kinematically, but amplitudes of dipping events are not preserved.

In our derivation, we have followed a similar approach in order to obtained the correct amplitude and phase spectrum in the Fourier domain using the stationary phase approximation essentially based on Rocca and Deregowski algorithm (1981). It is shown in the following section that stationary phase approximation gives the correct results in terms of both phase and amplitude spectrums.

Our result based on a distinct algorithm is the same as the result reached by Black *et al.* (1993). Consequently, our study is an option to gain insight into the mathematical and physical aspects of DMO from a different viewpoint and it is our belief that this derivation may be easily followed and comprehended.

THEORY

With the impetus given by the pioneering work of Deregowski and Rocca, (1981) integral DMO method we specifically, mention the excellent study of Black et al. (1993) in an effort to unravel the amplitude distribution along the DMO operator. In fact, they accurately established the dynamic behavior of the Operator both in space and Fourier domain. Same objective may be reached starting from Deregowski-Rocca Convolution integral, following the steps given below.

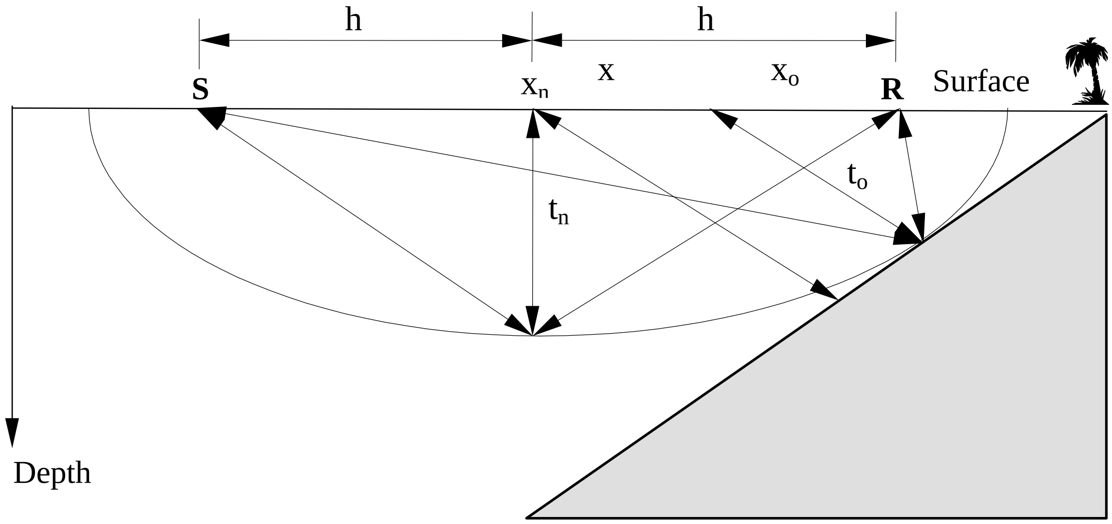

FIG.1. DMO ray path geometry for finite-offset and zero-offset, various travel times.

The general form of the Deregowski-Rocca integral (Deregowski and Rocca, 1981) may be written as follows:

$$P_o(t_o, x) = \iint P_n(t_n, x_n) s(t_o, x) dt_n dx_n \tag{1}$$

where

$$s(t_o, x) = \delta\left[t_o - t_n\left(1 - \frac{x^2}{h^2}\right)^{1/2}\right] \tag{2}$$

P_n is the NMO corrected data, t_n is the NMO corrected time, t_o is the zero-offset time, x is the zero-offset position with respect to the midpoint, x_n is the midpoint position on the NMO corrected section and h is the half offset as sketched in Figure 1.

The transform of s (t_o, x) into $s(\omega_o, k)$ domain carried out by Deregowski and Rocca (1981, page 398) yields.

$$s(\omega_o, k) = \frac{h(\pi)^{1/2}}{(2i\omega_o t_n)^{1/2}} e^{i\left[\omega_o t_n + \frac{k^2 h^2}{2\omega_o t_n}\right]} \tag{3}$$

The above formula differs from our solution.

For $x_o - x_n = x$ (see Fig. 1) and $t_n = t_o A$ (Sincer *et al*. 1993). The above equation may be reformulated in such a way that evaluations can be carried out with respect to t_o and x instead of t_n and x_n**.** When it is done, we end up with the following:

$$P_o\left(\frac{t_n}{A}, x\right) = \iint P_n(At_o, x_n) J s(t_o, x) dt_o dx \tag{4}$$

where

$$J = \begin{vmatrix} \frac{\partial t_n}{\partial t_o} & \frac{\partial x_n}{\partial t_o} \\ \frac{\partial t_n}{\partial x_o} & \frac{\partial x_n}{\partial x_o} \end{vmatrix} = \begin{vmatrix} \frac{A^3(x)}{2A^2(x)-1} & 0 \\ 0 & 1 \end{vmatrix}$$

$$J = \frac{A^3(x)}{2A^2(x)-1} \tag{5}$$

$$A(x) = \left(1 - \frac{x^2}{h^2}\right)^{-1/2} \tag{6}$$

We, now, consider the new operator

$$S(t_o, x) = \frac{A^3(x)}{2A^2(x)-1} s(t_o, x) \tag{7}$$

which is slightly different than the operator given by Black *et al.* (1993) in the sense that s (t_o, x) has been already defined by Deregowski and Rocca (1981) as smear stack operator. We only establish S(t_o, x) by taking into account the Jacobian $A^3(x)/2A^2(x)–1$ which result from the new integral definition which is to be transformed into the Fourier domain.

S(ω_o, k) may be Fourier transformed without serial expansion with respect to x.

Combining the ideas given above the following equation may be written

$$S(\omega_o, k) = \int \frac{A^3(x)}{2A^2(x)-1} \int\left[\delta\left(t_o - t_n\left(1 - \frac{x^2}{h^2}\right)^{1/2}\right) e^{i\omega_o t_o} dt_o\right] e^{-ikx} dx \tag{8}$$

as

$$\int\left[\delta\left(t_o - t_n\left(1 - \frac{x^2}{h^2}\right)^{1/2}\right) e^{i\omega_o t_o} dt_o\right] = e^{i\left[\omega_o t_n\left(1 - \frac{x^2}{h^2}\right)^{1/2}\right]}$$

$$S(\omega_o, k) = \int \frac{A^3(x)}{2A^2(x)-1} e^{i\left[\omega_o t_n\left(1 - \frac{x^2}{h^2}\right)^{1/2} - kx\right]} dx \tag{9}$$

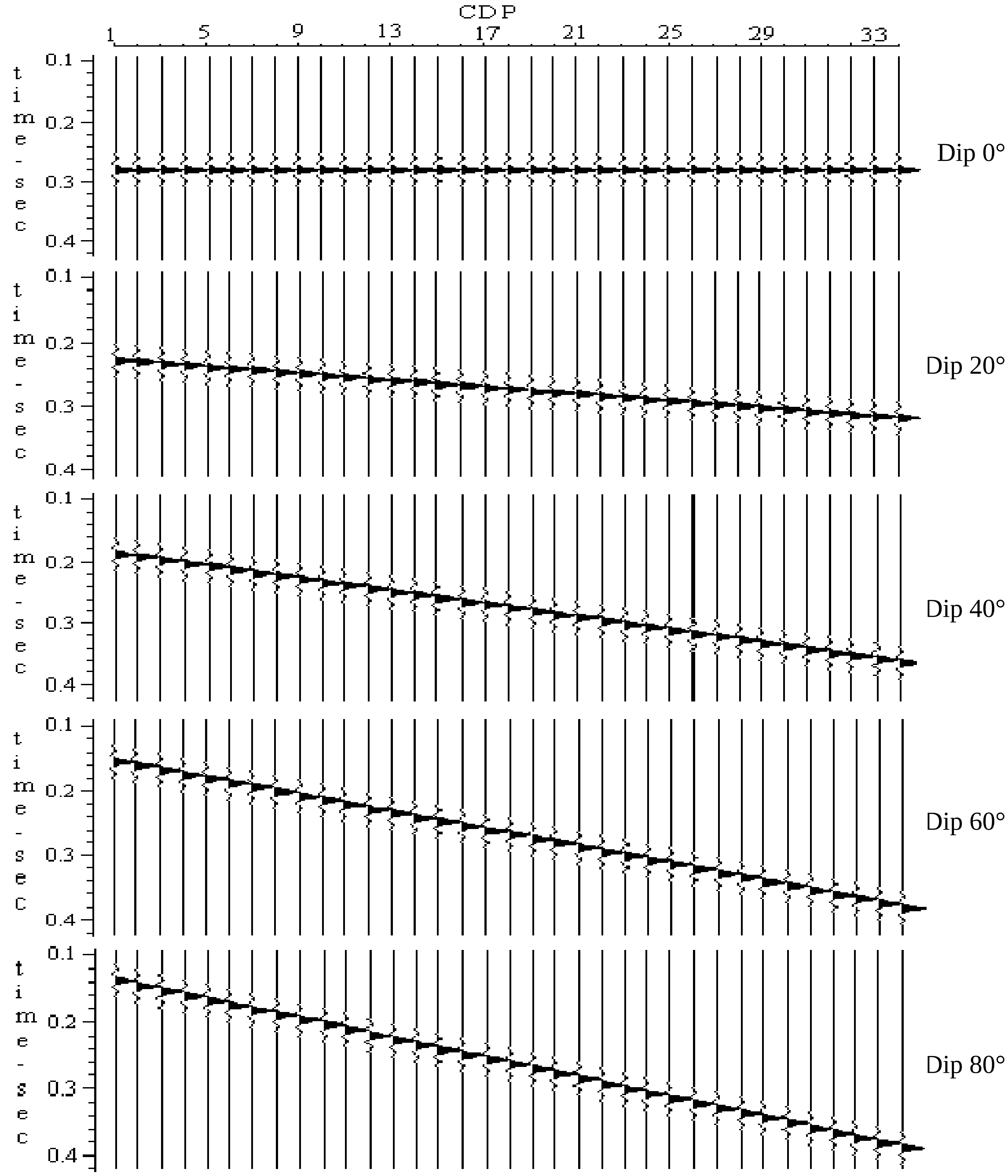

FIG. 2. Stacked gathers after NMO correction. Parameters for the data are V=3000 m/s and offset=850 m.

Any integral of the type $\int B(x)e^{if(x)}dx$ may be performed using the stationary phase method. The result will be (Newton, 1966; Bleistein and Handelsman, 1975)

$$\int B(x)e^{if(x)}dx = \frac{B(x_o)e^{if(x_o)}(2\pi i)^{1/2}}{[f''(x_o)]^{1/2}} \tag{10}$$

Where x_o represents the stationary point on the x-axis, $f(x_o)$ is the value of the phase at the stationary point and $f''(x_o)$ is the value of the second derivative at x_o.

Applying the same formalism to the integral (9) we find

$$S(\omega_o,k) = \frac{hA}{2A^2-1}\left(\frac{2\pi}{i\omega_o t_o}\right)^{1/2} e^{i\omega_o t_n A} \tag{11}$$

Stationary point value x_o and $f(x_o)$ are

$$f'(x) = \frac{\partial f(x)}{\partial x} = \frac{\partial}{\partial x}\left[\omega_o t_n\left(1-\frac{x^2}{h^2}\right)^{1/2} - kx\right] \tag{12}$$

$$f'(x) = 0$$

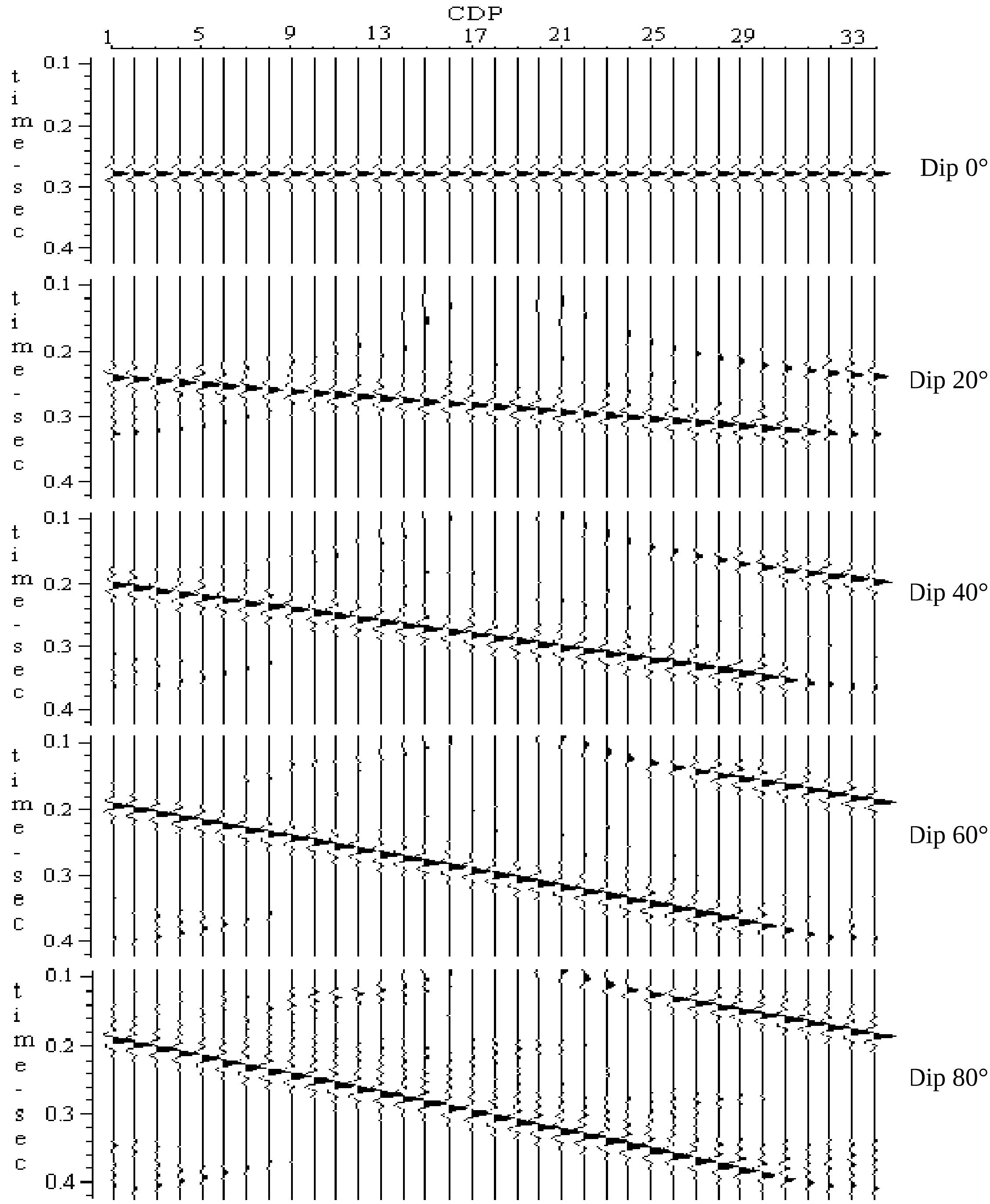

FIG. 3. DMO output of the data given in Figure 2 (True amplitude processing has been applied). Parameters for the data are V=3000 m/s and offset=850 m.

$$x_o = \pm \frac{kh^2}{\left(k^2h^2 + \omega_o^2 t_n^2\right)^{1/2}} \tag{13}$$

$$f(x_o) = \omega_o t_n A(x_o) \tag{14}$$

where $A(x_o)$ is equal to Hale’s quantity A, explained in k domain (Sincer *et al.* 1993)

$$A = \left(1 + \frac{k^2h^2}{\omega_o{}^2 t_n^2}\right)^{1/2} \tag{15}$$

The phase we have obtained here is exactly the phase value having are elliptical impulse response Hale originally found when he had driven his equation in 1984. In other words, his phase formula was correctly obtained, however, there was a flow in terms of amplitude spectrum. Therefore, in this study we will focus on the amplitude spectrum essentially.

$$f''(x_o) = -\frac{\omega_o t_n}{h^2} A^3 \tag{16}$$

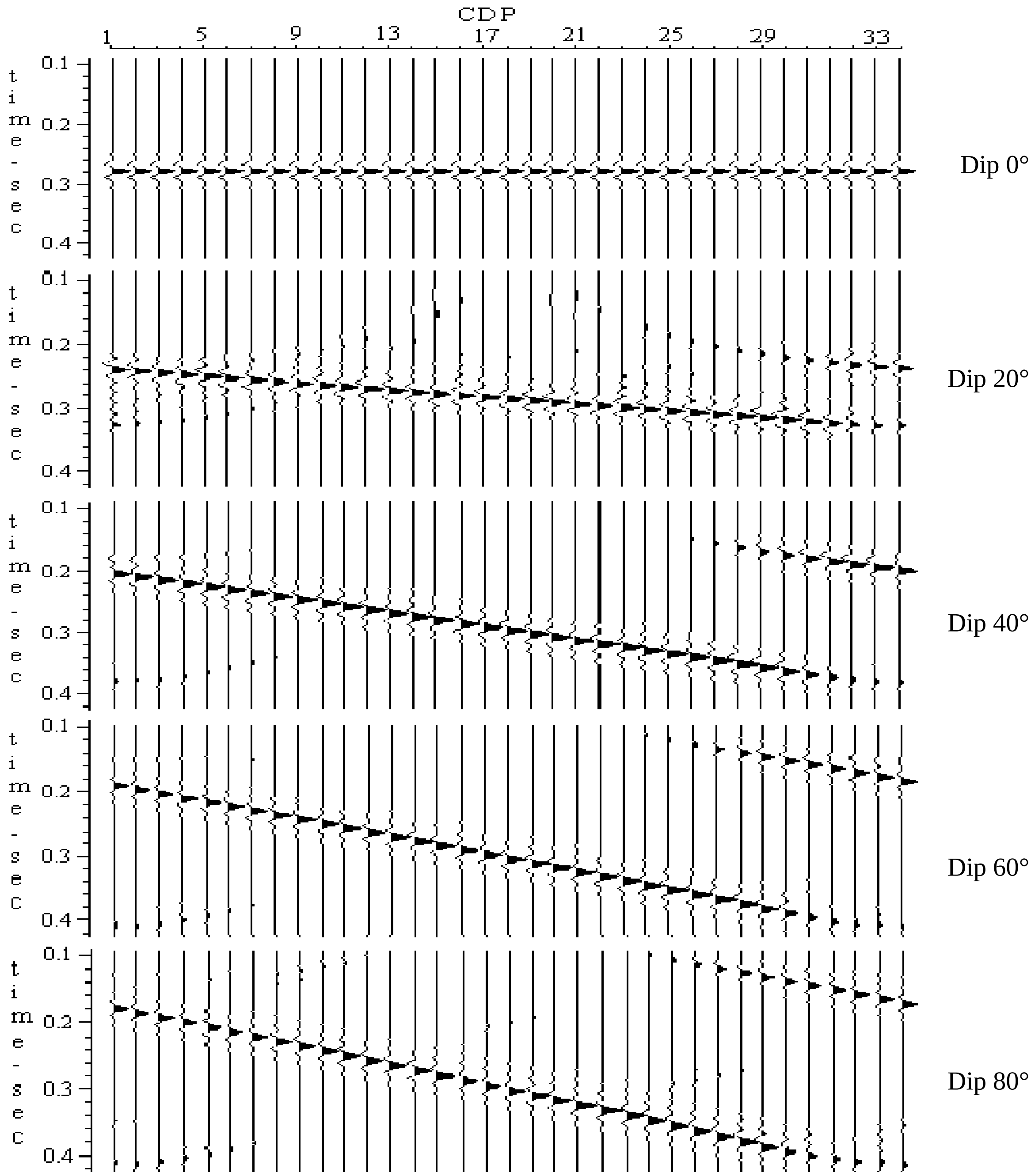

FIG.4. Processing of the same data given in Figure 2. with D.R. DMO operator.

On the other hand as

$$t_n = t_o A \tag{17}$$

and consequently,

$$\omega_n = \frac{\omega_o}{A} \tag{18}$$

may be derived and the following transformation may be written.

$$P_n(t_n) = P_n(t_o A) \tag{19}$$

$$P_n(t_o A) \Leftrightarrow \frac{1}{A} P_n\left(\frac{\omega_o}{A}\right) \tag{20}$$

Notice the change in ω_o content in the Fourier domain.

Using (11), first $P_o(\omega_o, k)$ and then by inverse Fourier transform $P_o(t_o, x)$ may be calculated.

$$P_o(\omega_o, k) = S(\omega_o, k) \frac{1}{A} P_n\left(\frac{\omega_o}{A}, k\right) \tag{21}$$

The importance of the following result may be stressed.

$$\frac{1}{A} S(\omega_o, k) = \frac{h}{2A^2 - 1} \left(\frac{2\pi}{i\omega_o t_o}\right)^{1/2} e^{i\omega_n t_n A} \tag{22}$$

This is definite Fourier domain operator, which is identical to the function $G(\omega_o)$ in the equation

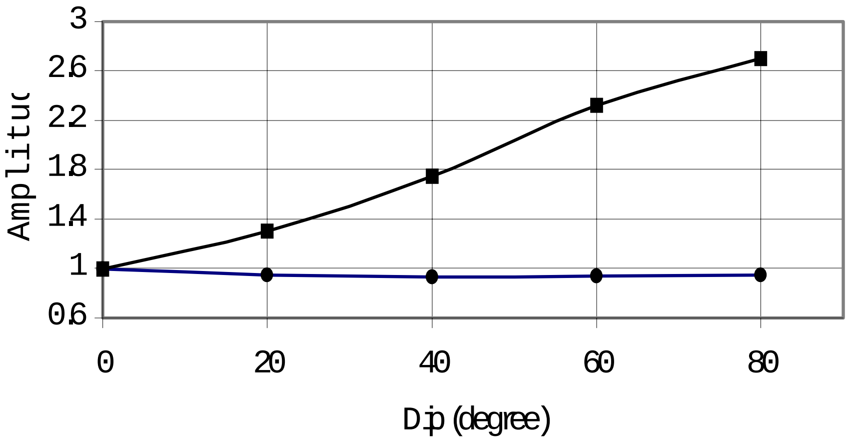

FIG.5. Plotted points with the circles and the squares are peak amplitudes obtained from the Figure 3 and 4 respectively. Amplitudes are normalized with respect to zero degree dip data. Parameters for the data are V=3000 m/s and offset=850 m. Comparison of these curves indicates that when amplitudes are preserved. There is a relatively small change with respect to dip-angle.

$$P_o(\omega_o, y_o, h) = Y(\eta_o)G(\omega_o) \quad (23)$$

where $Y(\eta_o)$ is Fourier transformed NMO data, given on page 54 of Black *et al*. (1993).

The amplitude part of the operator $\frac{1}{A}S(\omega_o, k)$ may be considered to be consisting of three multiplied terms. These are,

$$\frac{1}{A},\ \frac{A^3}{2A^2-1},\ \frac{h}{A}\left[\frac{2\pi}{i\omega_o t_o}\right]^{1/2}$$

Which are the contributions of the equation (20), the Jacobian and the phase-related term respectively.

Considering Dirac behavior of s(t_o, x), it is easy to show that $h\left[\frac{2\pi}{i\omega_o t_o}\right]^{1/2}$ must be equivalent, in the f-k domain, to A^3 as already observed, in a different way, by Black *et al*. (1993) while exact solution given by (22) corresponds, in the f-k domain, to $A^3/(2A^2-1)$. We may now compare, starting from a synthetic example, the results obtained from equations (1) and (22).

Figure 2 shows the NMO-corrected gathers for various dipping horizons used as input data to DMO operation associated with the parameters specified in the figure caption.

In obtaining these results the NMO corrected data $P_n(t_n, x_n)$ are transformed into the f-k domain $P_n(\omega_n, k)$ and multiplied by the inverse of $\frac{1}{A}S(\omega_o, k)$, previously calculated to obtain $P_o(t_o, x)$.

Figure 3 illustrates the inverse Fourier transform of the DMO applied data giving us the desired zero-offset section $P_o(t_o, x)$.

Figure 4 illustrates the results of the Deregowski-Rocca operator applied to the same input data.

Figure 5 shows the normalized amplitudes with respect to the value at zero angles for both the new and Deregowski-Rocca operators as indicated in squares and circles respectively. Obviously, when the new operator is implemented no significant amplitude variations are observed with respect to dip angle.

CONCLUSIONS

In terms of constant acoustic impedance values, no amplitude variation is expected for zero-offset data. Comparing the two curves shown in Figure 5 it is obviously seen that amplitudes values belonging to the new operator is approximately dip-independent. However, Deregowski-Rocca amplitudes don’t satisfy the same criteria.

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