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Bayesian extreme values distribution for seismicity parameters assessment in South America

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Abstract*: Seismicity parameters in South America are estimated with the application of Bayesian statistics. The distribution is named "distribution of extreme values", because in order to compute the parameters of seismicity, the distribution of maximum magnitude (i.e. an extreme value of magnitude in a given time span) is used. A concise analysis of the theory applied is given. Some simple and generally accepted hypotheses, like the Poisson model for the temporal distribution of earthquakes, as well as the exponential one for the distribution of the magnitudes of the earthquakes are adopted. Starting with these initial hypotheses and by means of Bayesian statistics, the distribution of the maximum expected magnitude for a given time span and the expected mean return period for a given magnitude are assessed. This is done in two steps. Firstly the temporal distribution of earthquake is found, while in the second step their magnitude's distribution is determined. Then the parameters of the latter distributions are estimated. According to the methodology of Bayesian statistics, in order to start with, prior values are assigned to the parameters. Utilising the conjugate distribution theory, the posterior values are estimated using the prior values and the observational data. As these posterior parameters do not have an obvious physical meaning, they are transformed to four familiar parameters of seismicity. These are the rate of occurrence of earthquakes, λ, the parameter of the distribution of magnitude, β, and their respective standard deviations. Seismicity parameters estimated and are expressed in terms of the probability of exceeding of a given magnitude in a given time span, and the mean return period of a given magnitude. These parameters are useful for any seismic hazard assessment in this area.*

Keywords: *Extreme-values, Bayesian Estimators, Seismicity Parameters, South America.*

INTRODUCTION

The theory of extreme value distributions was established by Gumbel (1958). In fact, the distribution used in the present study is similar to the distribution of Gumbel's first asymptote. Since its early years the extreme values theory had successfully been used for solving seismological problems (Milne and Davenport, 1965; Epstein and Lomnitz, 1966; Yegulalp and Kuo, 1974; Makropoulos and Burton, 1985, Tsapanos and Burton, 1991). Its advantages according to Lomnitz (1974), are: 1) no detailed knowledge of the distribution is needed except for its behavior near its upper end, 2) the extreme values are usually better known, more homogeneous and better defined compared to an average event and 3) its application is simple and does not require many assumptions.

Bayesian probability theory has two properties that make it valuable in the estimation of parameters related to the seismicity of an area. The first property provides a rigorous means of combining prior information on seismicity whether it is judgmental, geological or statistical with historical observations of earthquake occurrences. Such prior information may be used to supplement seismicity data when they are incomplete or inaccurate or cover a short time span. The second aspect provides a means of incorporating the statistical uncertainty associated with the estimation of the parameters used to quantify seismicity in addition to the probabilistic uncertainty associated with the inherent randomness of earthquake occurrence.

Campbell (1982) combined the extreme value theory with Bayesian statistics and later (Campbell, 1983) based on this combination he assessed the seismicity parameters of the San Jacinto fault zone. A similar procedure has been applied by Stavrakakis and Tselentis (1987) for a probabilistic prediction of strong earthquakes in Greece. Stavrakakis and Drakopoulos (1995) adopted the Bayesian extreme-value distribution of earthquake occurrence in order to estimate the seismic hazard in some seismogenic zones in Greece and the surrounding area

A Bayesian approach was applied by Tsapanos *et al*. (2001) for estimating the seismic hazard parameters in some regions of the circum-Pacific rim. More recently (Papoulia *et al*., 2001) applied the Bayesian extreme values distribution for the estimation of strong earthquakes in Messiniakos fault zone (southern Greece) based on seismological and geological data.

This paper confines itself to the Bayesian extreme-value distribution that applied in the seismological data of South America for the assessment of seismicity parameters. The choice of prior estimates concerning parameters, which are of physical meaning, like the rate of occurrence of earthquakes, λ and the parameter of the distribution of magnitudes, β, with their respective standard deviations.

The data set used, in the present study, covers a time span of 103 (1894-1996) years and is extracted from the World Earthquake database maintained by the British Geological Survey (Henni *et al*., 1998). The obtained catalogue was purged of foreshocks and aftershocks prior to analysis. Magnitudes equal or greater than 6.5 (of shallow focal depths, h<60 Km) are only taken under consideration for the present analysis because heavy damages and injuries or/and deaths, often were caused by such magnitudes and the examined area badly experienced of such earthquakes from time to time. The whole area is separated in six pre-define zones (Tsapanos, 2000). The epicenters of the earthquakes used in this study and the zones in which south America is divided are depicted in Figure 1.

ASSESSMENT OF THE EXTREME VALUES DISTRIBUTION PARAMETERS

Extreme values distribution

For the temporal distribution of earthquakes the Poisson model is assumed. The probability of n events occurring in time t is:

$$P(N = n \mid \lambda, t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \tag{1}$$

where λ is the positive parameter of the distribution. Following the Bayesian methodology we consider λ as a random variable. This means that λ can have any (positive) value with varying probabilities. Then the updated probability of n earthquakes occurring in time t is:

$$\tilde{P}(N = n \mid t) = \int_0^\infty P(N = n \mid \lambda, t) f(\lambda) d\lambda \tag{2}$$

where $f(\lambda)$ is the probability density function of λ.

It can be assumed that this parameter is Gamma-distributed. This assumption is reasonable enough and is not too limiting due to the variety of shapes that the Gamma distribution can have.

$$f(\lambda) = \frac{t''^{n''}}{\Gamma(n'')} \lambda^{n''-1} e^{-\lambda t''} \tag{3}$$

where t'', n'' are the posterior parameters of the Gamma distribution and Γ is the Gamma function. Substituting (3) in (2) and performing the integration it is finally found that:

$$\tilde{P}(N = n \mid n'', t'', t) = \frac{\Gamma(n + n'')}{n!\Gamma(n'')} \left(\frac{t''}{t + t''} \right)^{n''} \left(\frac{t}{t + t''} \right)^{n} \tag{4}$$

which yields the probability of n earthquakes occurring in t years taking in to account the uncertainty concerning the Poisson distribution parameter, that is the uncertainty about the rate of occurrence. In terms of Bayesian statistics theory and more specifically conjugate distributions, the prior values t' and n' are related to the posterior t'' and n'' by the following relations:

$$n'' = n_0 + n' \tag{5}$$

$$t'' = t_0 + t' \tag{6}$$

where n_0 and t_0 are the number of earthquakes that occurred during the observation period and the duration of this period, respectively. The relations between the Gamma distribution parameters and its mean and standard deviation are:

$$\bar{\lambda}' = \frac{n'}{t'} \tag{7}$$

$$\sigma_{\lambda'} = \frac{n'}{(t')^2} \tag{8}$$

Using these, then equations (5) and (6) can be written as:

$$n'' = n_0 + \left(\frac{\bar{\lambda}'}{\sigma'_\lambda} \right)^2 \tag{9}$$

$$t'' = t_0 + \frac{\bar{\lambda}'}{(\sigma'_\lambda)^2} \tag{10}$$

where $\bar{\lambda}'$ is the mean of the prior distribution of λ and σ'_λ its standard deviation.

Because the distribution of equation (4), was calculated using the Bayes theorem with the main variable (N) assumed to be Poisson distributed with a parameter assumed to be Gamma distributed it is called Bayes-Poisson-Gamma distribution. Figure 2 (a and b) are graphs of the Bayes-Poisson-Gamma distribution for two values of uncertainty about λ. This uncertainty is expressed as the standard deviation of the product λt. It is observed that if the uncertainty is low then the Bayes-Poisson-Gamma distribution approximates the Poisson distribution whereas if the uncertainty is high deviations are observed.

Similar considerations apply for the magnitude distribution of earthquakes. The probability density function of magnitudes of earthquakes with $M \geq m_l$ is:

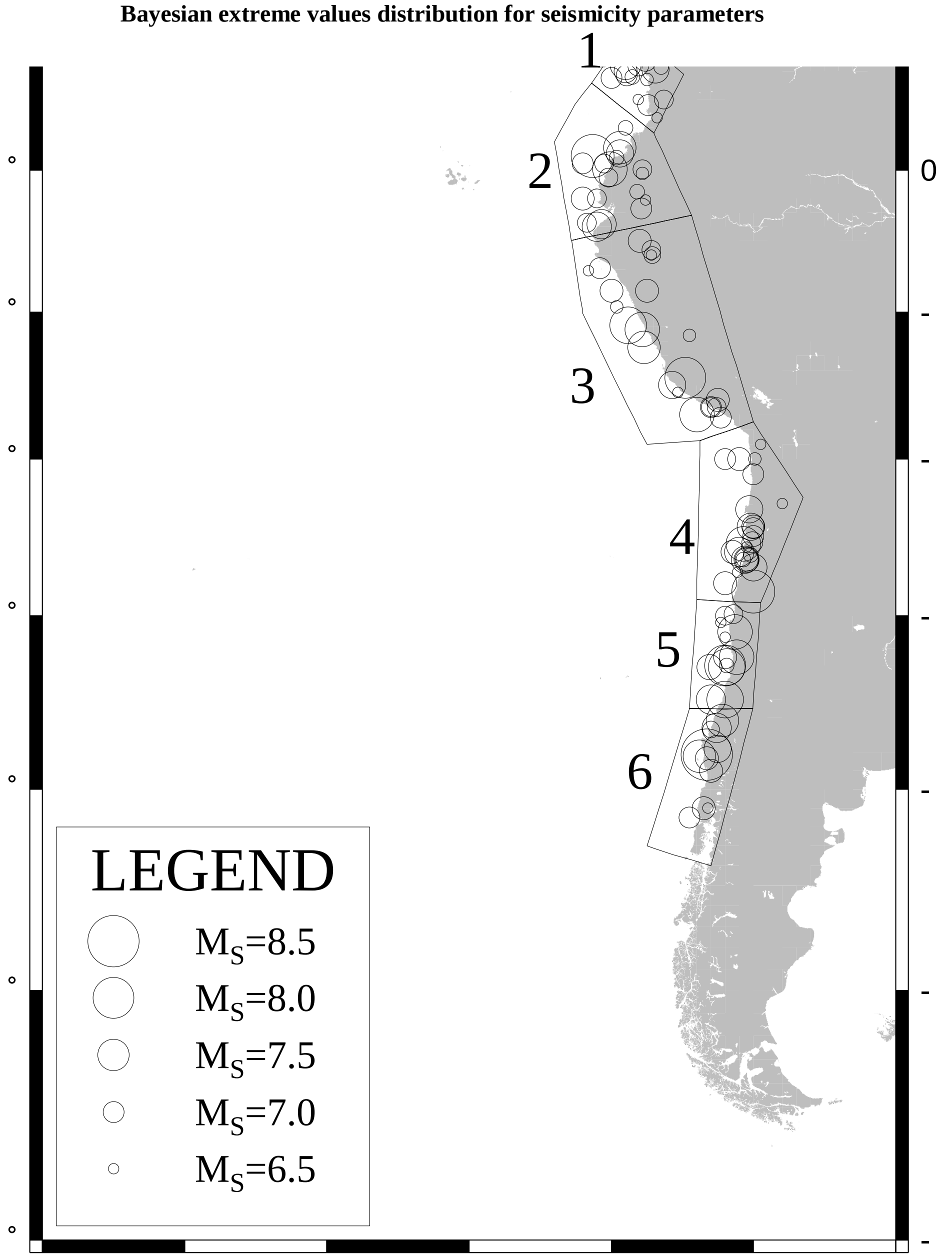

FIG. 1. The six examined zones of south America and the epicenters of earthquakes used in this study.

$$f(m) = \beta \exp[-\beta(m - m_1)] \qquad (11)$$

that is, the quantity (m-m_l) has the exponential distribution with parameter β. Parameter β is related to the parameter b of the Gutenberg-Richter law by $\beta = b$ ln10. This exponential distribution is compatible with the Gutenberg-Richter law (Epstein and Lomnitz, 1966).

The (cumulative) probability function of the magnitude distribution is particularly useful and is given by:

$$F(m \mid \beta, m_1) = P(M \le m \mid \beta, m_1) = 1 - \exp[-\beta(m - m_1)] \qquad (12)$$

Like the parameter λ, parameter β is also assumed to be a random variable. So, by means of Bayesian statistics, the probability function can be written as:

$$\tilde{F}(m \mid m_1) = \int_0^\infty F(m \mid \beta, m_1) f(\beta) d\beta \qquad (13)$$

where $f(\beta)$ is the probability density function of the distribution of β. In a similar manner with the λ parameter, we can assume that β has the Gamma distribution:

$$f(\beta) = \frac{m''^{\gamma''}}{\Gamma(\gamma'')} \beta^{\gamma''-1} e^{-\beta m''} \qquad (14)$$

where m'' and γ'' are the posterior parameters of the Gamma distribution. Substituting in *(13)*, we come up with:

$$\tilde{F}(m \mid m_1) = \begin{cases} 1 - \left(\dfrac{m''}{m'' + m - m_1} \right)^{\gamma''} & m_1 \le m \le \infty \\ 0 & m \le m_1 \end{cases} \qquad (15)$$

According to this, the function is equal to unity for infinite magnitude. In reality there is a physical upper limit in magnitude, m_u. Normalising *(15)* so that it is equal to unity for magnitude equal to m_u, it becomes:

$$\tilde{F}(m \mid m_l, m_u) = \begin{cases} 1 & m_u < m \\ K\left[1-\left(\dfrac{m''}{m''+m-m_l}\right)^{\gamma''}\right] & m_l \le m \le m_u \\ 0 & m < m_l \end{cases} \quad (16)$$

where:

$$K = \left[1-\left(\frac{m''}{m''+m_u-m_l}\right)^{\gamma''}\right]^{-1} \quad (17)$$

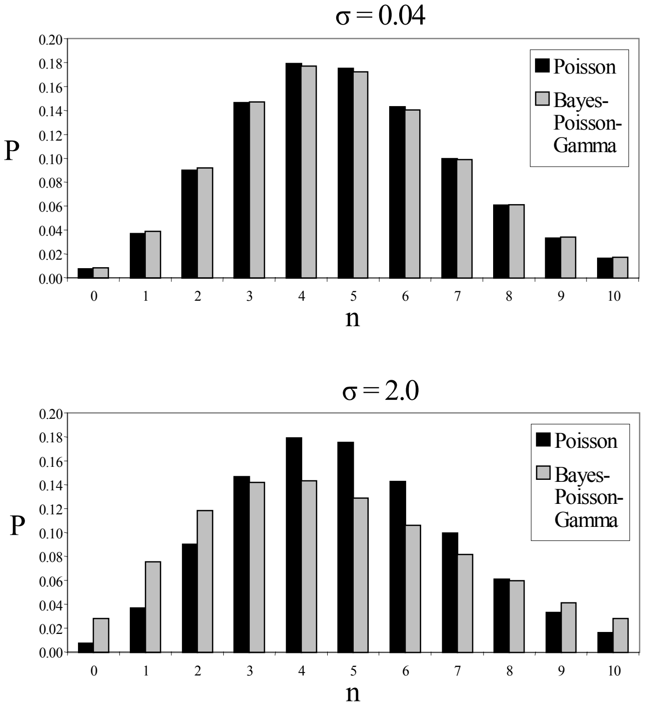

FIG. 2. The Poisson distribution compared to Bayes-Poisson-Gamma one for: a) low value of uncertainty and b) large value of uncertainty, concerning parameter λ

In Figure 3, the graphs of the functions described by (15) and (16) for an upper bound magnitude equal to 8.5, are depicted. For most normal values the parameters of the two function are very close to each other, as can be seen in the graph. In the present work relation (16) was preferred, although it is difficult to determine an upper limit for magnitude, because of the physical meaning of the relation. The upper limits in magnitude, for each zone, as it is computed by the maximum likelihood method (Kijko and Sellevoll, 1989), are listed in Table 1.

After the determination of form of the distribution of magnitudes, specific posterior values for the parameters can be computed. It was shown that the main variable, m-m_l, has an exponential distribution with parameter β, and that parameter β in its turn follows the Gamma distribution. By the conjugate distribution theory, the relations between prior and posterior parameters of the Gamma distribution are:

$$\gamma'' = \gamma_0 + \gamma' \quad (18)$$

$$m'' = \sum (m - m_l) + m' = n_0(\overline{m} - m_l) + m' \quad (19)$$

Given that (in Gamma distribution) the mean is $\mu = \dfrac{\beta}{a}$ with a dispersion $\sigma^2 = \dfrac{\beta}{\alpha^2}$ the latter by means of the μ and σ^2 can be written as:

$$\gamma'' = \gamma_0 + \left(\frac{\overline{\beta}'}{\sigma'_\beta}\right)^2 \quad (20)$$

$$m'' = n_0(\overline{m} - m_l) + \frac{\overline{\beta}'}{(\sigma'_\beta)^2} \quad (21)$$

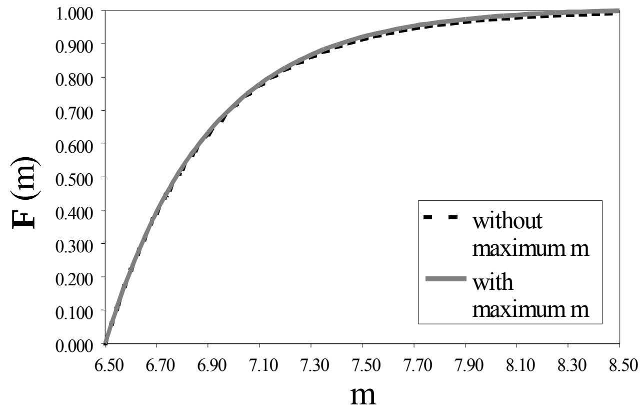

FIG. 3. Cumulative probability function of magnitude, where with continue line the magnitude with maximum *m* is presented, while the dashed line the magnitude without maximum *m*, is illustrated.

Table 1. The upper bound magnitude M_{max} for the six zones of South America. Prior values of λ and β and their standard deviations σ_λ and σ_β respectively.

Zone	M_{max}	λ (for $M \geq 6{,}5$)	σ_λ	β	σ_β
1	7.6	0.102	0.021	1.980	0.437
2	8.2	0.155	0.030	1.727	0.230
3	8.1	0.252	0.033	2.003	0.322
4	8.2	0.475	0.061	1.405	0.253
5	8.2	0.445	0.064	1.266	0.207
6	8.7	0.170	0.035	1.474	0.299

Equations (4) and (16) fully describe the temporal and magnitude distribution of earthquakes. From these, any parameter of seismicity can be computed. The distribution of extreme values of earthquake magnitude is particularly interesting. From the above relations and the total probability theorem it is concluded that the probability of exceeding of magnitude *m* in time *t* is given by:

$$\widetilde{P}(M_{max} \geq m \mid t) = \sum_{n=0}^{\infty} \widetilde{P}(N = n \mid n'', t'', t) \widetilde{F}(m)^n \quad (22)$$

After summing up the infinite terms in the right hand part (equation 22), it becomes:

$$\widetilde{P}(M_{max} \geq m \mid t) = 1 - \left(\frac{t''}{t'' + t[1 - \widetilde{F}(m)]} \right)^{n''} \quad (23)$$

where $\widetilde{F}(m)$ is the function given by (16) (or 15 if there is no upper limit in magnitude). Additionally, the mean return period of magnitude m is given by:

$$T_m = \frac{1}{\bar{\lambda}''_m} \quad (24)$$

where $\bar{\lambda}''_m$ is the occurrence rate for magnitude m or greater. Given that earthquakes are Poisson-distributed with parameter $\bar{\lambda}''$, then earthquakes of magnitude m or greater are Poisson-distributed with parameter $p\bar{\lambda}''$, where p is the probability of an earthquake to have a magnitude greater than or equal to m (Benjamin and Cornell, 1970). Since $p = 1 - \widetilde{F}(m)$, it follows that $\bar{\lambda}''_m = \bar{\lambda}''[1 - \widetilde{F}(m)]$. And because $\bar{\lambda}'' = n''/t''$, we obtain:

$$T_m = \frac{t''}{n''[1 - \widetilde{F}(m)]} \quad (25)$$

The relations (23) and (25) are the ones that were used for the determination of the parameters of seismicity in the six zones of South America.

Prior values of the parameters

In Bayes statistics, the prior values of the parameters are of equal importance in comparison to the observational data. If there is low uncertainty on prior values then they determine the posterior values. If, in the contrary, there is high uncertainty on the prior values then the posterior values are determined by the observations.

The prior values of the two parameters used in the present case, which are needed, are the occurrence rate λ and the magnitude distribution parameter β. In past works, the prior values were assessed by empirical estimations (Egutchi *et al.*, 1979), geological data (Wallace, 1970; Sieh, 1977), seismotectonic data (Campbell, 1978, 1983; Anderson 1979; Molnar, 1979; Mortgat and Shah, 1979) or statistical processing of seismological data (Esteva, 1969; Lomnitz, 1969; Newmark and Rosenblueth, 1971; Papadopoulos, 1987, 1990).

The assessment of λ using seismotectonic data is done by empirical relations, which associate it with rupture zone parameters, as the rupture zone dimensions, the seismic slip rate and the shear modulus μ. In this case it is difficult to estimate the errors. Additionally, there are no reliable relations of this kind for β (Campbell, 1983).

For the assessment of prior values by means of seismological data, it is possible to use earthquake catalogues. One possibility is to use the catalogue of a broad area and let the parameters calculated be the prior parameters of individual sources. A second possibility is to use a catalogue consisting of a large number of earthquakes of smaller magnitudes.

Because of the aforementioned limitations in the use of seismotectonic data and because it was difficult to find homogeneous empirical or geological estimations for all the zones under study, the last alternative was preferred.

For the calculation of the parameters λ and β, their standard deviations, as well as the upper bound magnitude for each zone, the approach proposed by Kijko and Sellevol (1989) was used. This methodology allows the seismicity parameters to be calculated by inhomogeneous data, using the maximum likelihood criterion. The accuracy of the method is acceptable, since in the present study it is used for the assessment of prior values and not for final results.

The values of the parameters and their errors as were assessed are given in Table 1. In the first column, the name of the zone is appeared. In the second the upper bound magnitude (M_{max}) is written. In the third and the fourth columns, the earthquake occurrence rate, λ, that corresponds to the earthquakes under consideration and its standard deviation σ_λ, is presented. Finally, the fifth and sixth column, corresponds to the earthquake magnitude distribution parameter, β, and its standard deviation σ_β .

The prior parameters n', t', γ' and m' are simply the last terms of the right hand parts of the equations (9), (10), (20) and (21) respectively. These values are presented in Table 2.

Table 2. Prior parameters of the distributions of the occurrence times and the magnitudes.

Zone	n'	t'	γ'	m'
1	23.000	224.575	20.488	10.346
2	27.000	174.326	56.250	32.572
3	58.000	229.806	38.617	19.277
4	60.000	126.288	30.752	21.894
5	49.000	109.992	37.346	29.489
6	23.000	135.320	24.237	16.447

Transition from prior to posterior distributions

Transition from prior to posterior distributions is done by means of relations (5), (6), (18) and (19). The first term of each right hand part corresponds to the observed data. Their physical meaning is: they are the values that the posterior parameters will have if no prior estimation is taken into account (or the uncertainty of the prior estimations is infinite).

The values of the first terms as were calculated from the sample are presented in Table 3. For each zone, the parameters n_0, T_0, γ_0 and m_0, in this order, are shown. Their meaning is explained later within the text.

Table 3. Observed data used

Zone	n_0	T_0	γ_0	m_0
1	14	103	14	5.9
2	19	103	19	10.8
3	22	103	22	12.7
4	25	103	25	13.6
5	13	103	13	9.7
6	11	103	11	8.5

where n_0 is the number of earthquakes listed for each zone, and γ_0 has the same physical meaning with n_0 and this is why they have the same value. T_0 is the period covered by the observations. This period is the same for all zones, that is 103 years, from 1894 up to 1996. The quantity $m_0 = n_0(\overline{m} - m_l)$ is a measure of the magnitude distribution. More specifically:

$$\beta = 1/(\overline{m} - m_l) \qquad (26)$$

where β is the parameter of the magnitude distribution, $\overline{m}$ is the average magnitude of the n_0 earthquakes used in each zone and m_l the lower magnitude limit, that is 6.5. This means that m_0 is inversely proportional of β, as well as b.

RESULTS

The posterior parameters are computed using the Bayes theorem is given by equation:

$$f''(\theta) = \frac{P(\varepsilon \mid \theta) f'(\theta)}{P(\varepsilon)} = \frac{P(\varepsilon \mid \theta) f'(\theta)}{\int_{-\infty}^{+\infty} P(\varepsilon \mid \theta) f'(\theta) d\theta} \qquad (27)$$

where $f'(\theta)$ is the prior distribution of the parameter θ, $P(\varepsilon|\theta)$ is the likelihood of the observations ε given θ and $P(\varepsilon)$ is the probability of the observations ε over all θ.

As the appropriate conjugate distributions were used, the computations can be performed without the use of above equation and the integral in the denominator of its right hand part. This is the reason that the parameters λ, σ_λ, β and σ_β, were transformed to n', t', γ' and m'. The relation written above is reduced to an addition of Tables (2 and 3), according to equations (5), (6), (18) and (19). The posterior parameters as calculated by these equations are given in Table 4.

Table 4. Posterior parameters of the distributions.

Zone	n''	t''	γ''	m''
1	37.000	327.575	34.488	16.246
2	46.000	277.326	75.250	43.372
3	80.000	332.806	60.617	31.977
4	85.000	229.288	55.752	35.494
5	62.000	212.992	50.346	39.189
6	34.000	238.320	35.237	24.947

Table 5. Posterior values of λ and β and their standard deviations.

Zone	λ''	σ_λ''	β''	σ_β''
1	0.113	0.019	2.123	0.361
2	0.166	0.024	1.735	0.200
3	0.240	0.027	1.896	0.243
4	0.371	0.040	1.571	0.210
5	0.291	0.037	1.285	0.181
6	0.143	0.024	1.412	0.238

These four parameters will be used to describe the seismicity of the studied area, by substituting in equations (23) and (25). It is useful, though, to transform the parameters n″, t″, γ'' and m″ back to λ, σ_λ, β and σ_β and then examine their values. This is done in Table 5. In the first column the zone name is given. In the second and third the corresponding earthquake occurrence rate, λ and its standard deviation are tabulated, while in the fourth and fifth the parameter of the magnitude distribution, β and its standard deviation are presented.

Compared to their prior values as they are given in Table 1, the values of the parameters seem to be slightly altered. In the case of β, there is an increase in most cases. This alteration is caused by the incorporation of additional information. Nevertheless, the alteration is rather small. This is mainly because the prior parameters were calculated very accurately. In addition to this the prior parameters were estimated using the maximum likelihood method, which has some similarities with the Bayesian methodology. Finally, this might be explained by the fact that larger earthquakes ($M \geq 6.5$) which used for this study are more accurately determined. The values of standard deviations have decreased. This is because additional information that was incorporated in the model had as a result the decrease in uncertainty.

ESTIMATION OF SEISMICITY PARAMETERS

Probability of exceeding of magnitude m in 10, 20 and 75 years

The probability of exceeding of magnitude m in 10, 20 and 75 years is computed by equation (23) for t=10, 20 or 75. The probability of exceeding as a function of magnitude is mapped in Figure 4.

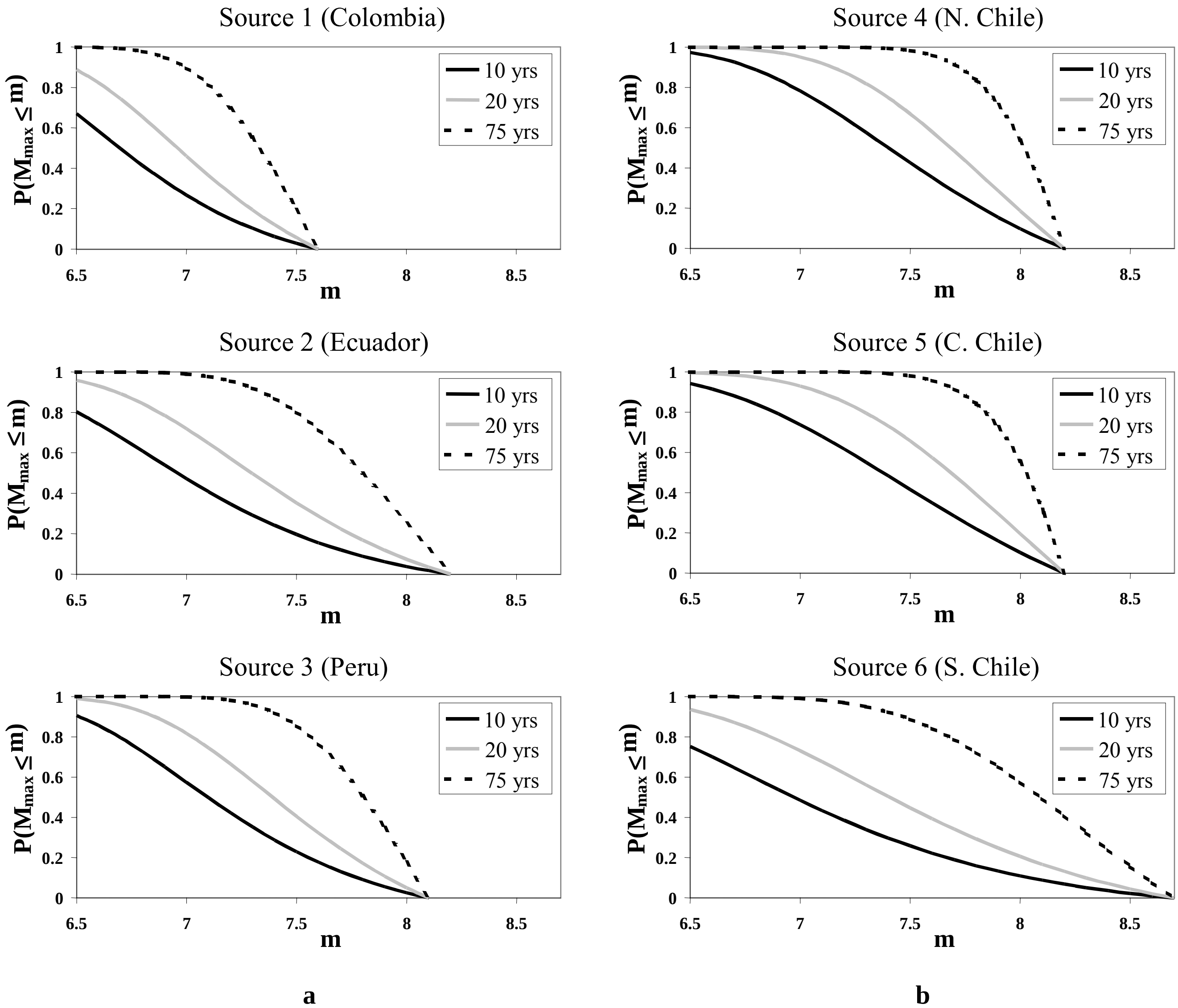

FIG. 4. Probabilities of exceeding of magnitude *m* for time periods of 10, 20 and 75 years: a) for the zones (1, 2 and 3) which correspond to Colombia, Equator and Peru and b) for the zones (4, 5 and 6) which are north, central and south Chile, respectively.

These probabilities, estimated by Bayesian statistics are more reliable than the ones evaluated by classical methods because: a) there is no subjective choice of a confidence interval since the uncertainty of the estimation is incorporated into the result b) they contain additional information incorporated into the model in a way which is consistent with the theory and c) they are easy to update in case that new data becomes available. In addition to these, it is of importance that the model makes use of an upper bound magnitude, which makes it more realistic.

The largest probabilities of exceeding are observed in zone 4 (North Chile, about 97% for t=10 years) and the smallest in zone 1 (Colombia, about 68% for t=10 years). The spatial distributions of the probabilities show a variation from one zone to another. The particularity of zone 6 (South Chile) is more visible. There is a high probability of a very strong earthquake to occur in the long run, out of proportion with occurrences of probabilities of smaller earthquakes. As mentioned before, the probabilities of occurrence of earthquakes with a magnitude equal to the upper limit are equal to zero, in accordance with the model.

In every zone there is a certainty of occurrence of strong earthquakes in the long run. In zones of high seismicity, it is certain even for magnitudes as large as 7.5, like in zones 4 and 5 (North and Central Chile).

Mean return period of magnitude m

The mean return period of an earthquake of magnitude M, is estimated by the relation (25). The usefulness of these results are: a) in case we are interested in a time period of a specific duration, e.g. the life time of a construction, we can determine the magnitude in which it corresponds, and b) if, inversely, we are interested in

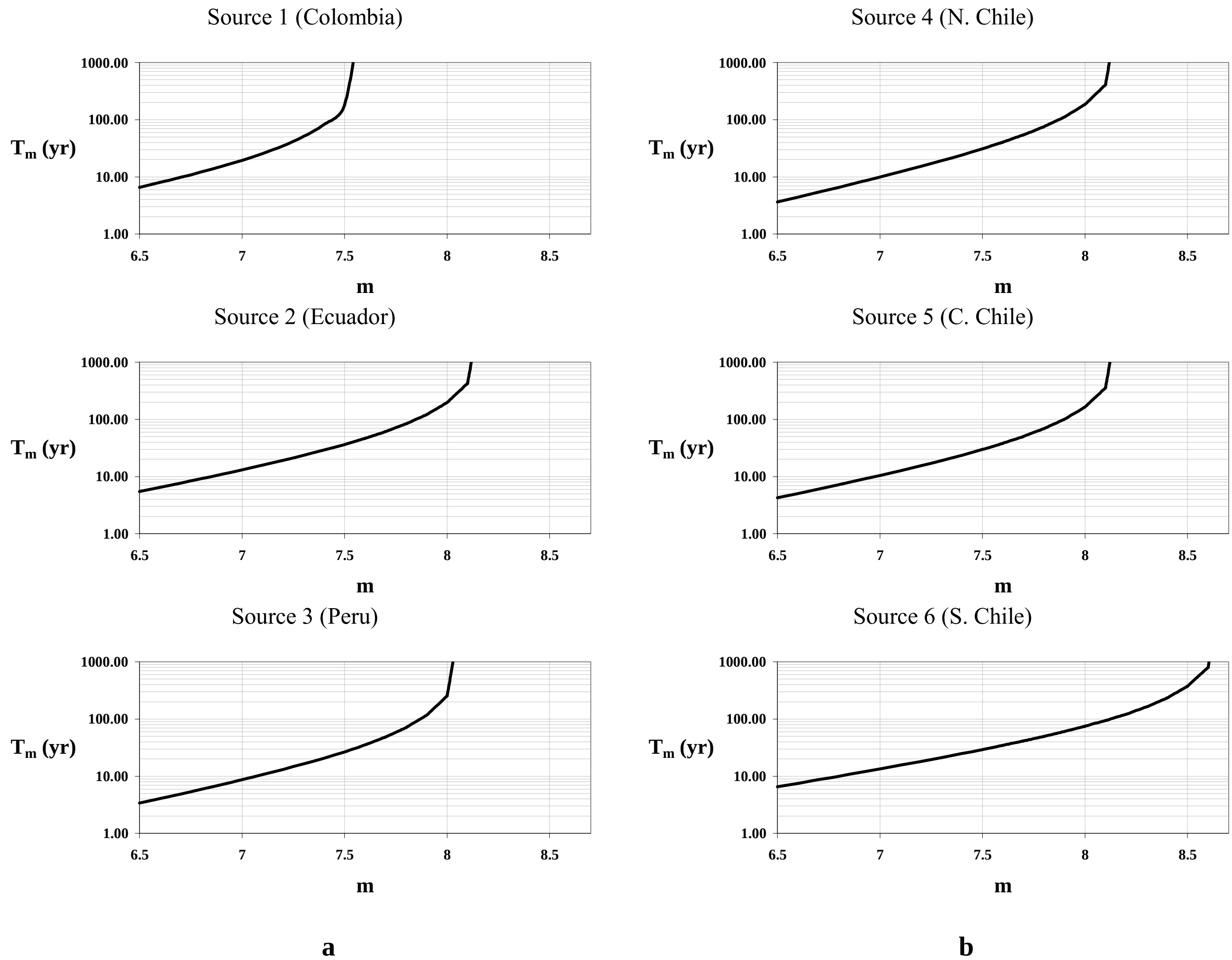

FIG. 5. Mean return period of magnitude m: a) for the zones (1, 2 and 3) which correspond to Colombia, Equator and Peru and b) for the zones (4, 5 and 6) which are north, central and south Chile, respectively.

a specific magnitude, e.g. if for a specific area, earthquakes are considered to be destructive when their magnitude is larger than *m*, then we can know the expected time interval between these earthquakes. The above examples are simplified but they demonstrate the importance of quantifying seismicity.

In Table 6, the mean return periods for magnitudes equal to 7.0, 7.5 and 8.0 are shown. Mean return period as a function of magnitude was mapped for every zone in Figure 5. For the mean return period, logarithmic scale was used.

Table 6. Mean return period T_m (in years) for magnitudes 7.0, 7.5 and 8.0.

Zone	$T_{7.0}$	$T_{7.5}$	$T_{8.0}$
1	19.5	179.8	---
2	13.1	36.5	197.5
3	8.7	26.7	254.3
4	9.9	31.1	185.8
5	10.4	29.8	164.6
6	13.4	29.3	74.7

For relatively small magnitudes, the logarithm of mean return period is a linear function of magnitude, as expected by the Gutenberg-Richter law. For magnitudes close to the upper bound magnitude, the curve bends upwards because of the existence of the limit. A finite return period can only be defined for magnitudes that can actually occur.

It is interesting to observe the mean return period for the maximum observed magnitude of each zone. This return period is of the same order of magnitude as the time period covered by the catalogue. This confirms the reliability of the results as it demonstrates that the observation period is long enough to contain at least one earthquake with a magnitude close to the upper magnitude limit of each zone.

For zone 6 (South Chile), it is observed that the mean return period for a magnitude 8.5 is approximately 400 years, which is much longer than the period of the observations used for the estimations. This may explain the particularity of this zone's seismicity. In addition to this, a return period of 400 years is in agreement with the research of Nishenko (1991) and Scholtz (1994) on earthquakes in the south part of South America.

CONCLUSIONS

In the present study we assessed the seismicity parameters for six zones in South America by means of the Bayesian extreme value distribution. From the analysis presented here we can conclude how important is the uncertainty in the Bayesian estimates of seismicity. The results also point out the significance of the existence of an upper bound magnitude on the evaluation of seismicity parameters.

Prior values of the parameters are estimated based on the approach introduced by Kijko and Sellevoll (1989), using for this purpose the maximum likelihood criterion. The difference between the values of the prior and posterior parameters are very small and we conclude that this is due on the very accurate estimation of the prior parameters, as well as on the data used ($M \geq 6.5$) which of course are better determined.

The seismicity parameters evaluated are the probability of exceeding of magnitude m in 10, 20 and 75 years. The first two time-periods are related with the long-term earthquake prediction (Nishenko, 1985; Papazachos *et al.*, 1987; Papazachos and Papaioannou, 1993) while the last one is chosen as the life time of the ordinary buildings. We conclude that the probabilities, estimated by Bayesian statistics are more reliable than the ones evaluated by classical methods because: a) there is no subjective choice of a confidence interval since the uncertainty of the estimation is incorporated into the result b) they contain additional information incorporated into the model in a way which is consistent with the theory and c) they are easy to update in case that new data becomes available. For t=10 years zone 4 (North Chile) illustrates the largest probability of exceeding, while the smallest is observed in zone 1 (Colombia). Two more zones 3 and 5 (Peru and Central Chile) show probabilities equal or greater than 90%. Zone 2 (Ecuador) has a probability of exceeding 80%, while in S. Chile this probability is 77%. For some of these zones Papadimitriou (1993) found probabilities for earthquakes ($M \geq 7.5$) occurrences within the next decade (1992-2002) based on time-dependent seismicity. We observed, for instance, that south Colombia show a probability of earthquake genesis 68%, for southern Peru is 81%, for central Chile is 63%, while for south Chile is 60%.

The other obtained parameter of seismicity is the mean return period of a given magnitude which are (7.0, 7.5 and 8.0) These magnitudes are chosen in the sense that the examined area often experienced such magnitudes. It is interesting that for magnitudes 7.0 and 7.5 Colombia demonstrates the lowest values. We observed that the mean return period for the maximum magnitude of each zone is of the same order of magnitude as the time period covered by the catalogue used. So we can conclude that this verifies the creditability of the results as it illustrates that the time span used is long enough to contain at least one shock with a magnitude close to the upper bound of each zone.

REFERENCES

Anderson, J. G., 1979. Estimating the seismicity from geological structure for seismic risk studies: Bull. Seismol. Soc. Am., **69**, 135-158.

Benjamin, J. R. and Cornell, C. A., 1970. Probability, Statistics and Decision for Civil Engineers: McGraw-Hill, New York, 684 p.

Campbell, K. W., 1978. An estimate of earthquake recurrence times from seismotectonic data on a fault. Geologic guide and Engineering geology case histories, Los Angeles Metropolitan area, First Annual California section Confer. Los Angeles, May 12-14 1978: Association of Engineering Geologists, Los Angeles California, 95-101.

Campbell, K. W., 1982. Bayesian analysis of extreme earthquake occurrences. Part I. Probabilistic hazard model: Bull. Seismol. Soc. Am., **72**, 1689-1705.

Campbell, K. W., 1983. Bayesian analysis of extreme earthquake occurrences. Part II. Application to the San Jacinto fault zone of southern California: Bull. Seismol. Soc. Am., **73**, 1099-1115.

Enguchi, R. T., Campbell, K. W. and Wiggins, J. H., 1979. A survey of expect opinion on active and potentially active faults in California, Nevada, Arizona and Northern Baja California: Technical report 79-1328-2, J. H. Wiggins company Redonlo beach, California.

Epstein, B. and Lomnitz, C., 1966. A model for the occurrence of large earthquakes: Nature, **211**, 954-956.

Esteva, L., 1969. Seismicity prediction: Bayesian approach. Proc. of the Forth World Conf. on Earthq. Eng., Santiago, Chile 1, 172-184.

Gumbel, E. J., 1958. Statistics of Extremes. New York, Columbia Univ. Press, 374 p.

Henni, P. H. O., Fyfe, C. J. and Marrow, P. C., 1998. The BGS world seismicity database: Technical Report, no WL/98/13 BGS, Edinburgh.

Kijko, A. and Sellevoll, M. A., 1989. Estimation of earthquake hazard parameters from incomplete data files. Part I. Utilization of extreme and complete catalogues with different threshold magnitudes: Bull. Seism. Soc. Am., **79**, 645-654.

Lomnitz, C., 1969. An earthquake risk map of Chile: Proc. of the Fourth World Conf. on Earthq. Eng., Santiago, Chile 1, 161-171.

Lomnitz, C., 1974. Global Tectonics and Earthquake Risk: Elsevier Scientific Publishing Company Amsterdam, 320 p.

Makropoulos, K. C. and Burton, P. W., 1985. Seismic hazard in Greece. I. Magnitude recurrence: Tectonophysics, **117**, 205-257.

Milne, W. G. and Davenport, A. G., 1965. Statistical parameters applied to seismic regionalization: Proc. of the Third World Conf. on Earthq. Engin., Auckland New Zealand 1, 181-193.

Molnar, P., 1979. Earthquake recurrence intervals and plate tectonics: Bull. Seismol. Soc. Am., **69**, 115-133.

Mortgat, C. P. and Shah, H. C., 1979. A Bayesian model for seismic hazard mapping: Bull. Seismol. Soc. Am., **69**, 1237-1251.

Newmark, N. M. and Rosenblueth, E., 1971. Fundamentals of Earthquake Engineering, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 640 p.

Nishenko, S. P., 1985. Seismic potential for large and great interplate earthquakes along the Chilean and southern Perouvian margins of south America: a quantitative reappraisal: J. Geophys. Res., **90**, 3589-3615.

Nishenko, S. P., 1991. Circum-Pacific seismic potential: 1989-1999: Pageoph, **135**, 169-259.

Papadimitriou, E. E., 1993. Long-term earthquake prediction along the western coast of south and central America based on a time predictable model: Pageoph, **140**, 301-316.

Papadopoulos, G. A., 1987. An alternative view of the Bayesian probabilistic prediction of strong shocks in the Hellenic arc: Tectonophysics, **132**, 311-320.

Papadopoulos, G. A., 1990. Deterministic and Stochastic models of the seismic and volcanic events in the Santorini volcano. Thera and the Aegean World Proc. of the Third Intern. Congr., (edt. D.A. Hardy) **2**, 151-158.

Papazachos, B. C. and Papaioannou, Ch. A., 1993. Long-term earthquake prediction in the Aegean area based on a time and magnitude predictable model: Pageoph, **140**, 593-612.

Papazachos, B. C., Papadimitriou, E. E., Kiratzi, A. A., Papaioannou, Ch. A, and Karakaisis, G. F., 1987. Probabilities of occurrence of large earthquakes in the Aegean and the surrounding area during the period 1986-2006: Pageoph, **125**, 597-612.

Papoulia, J., Stavrakakis, G. and Papanikolaou, D., 2001. Bayesian estimation of strong earthquakes in the inner Messiniakos fault zone, southern Greece, based on seismological and geological data: J. Seismology, **5**, 233-242.

Scholz, C. H., 1994. The Mechanics of Earthquakes and Faulting. Cambridge University Press, 439 p.

Sieh, K. E., 1977. A study of Holocene displacement history along the south-central reach of the San Andreas fault: PhD Dissertation, Stanford University, Stanford Calofornia, 219 p.

Stavrakakis, G. N. and Tselentis, G. A., 1987. Bayesian probabilistic prediction of strong earthquakes in the main seismogenic zones of Greece: Boll. Geofis. Teor. Applic, **113**, 51-63.

Stavrakakis, G. N. and Drakopoulos, J., 1995. Bayesian probabilities of earthquake occurrences in Greece and surrounding areas: Pageoph, **144**, 307-319.

Tsapanos, T. M., 2000. The depth distribution of seismicity parameters estimated for the south America area: Earth Planet, Sci. Lett., **180**, 103-115.

Tsapanos, T. M. and Burton, P. W., 1991. Seismic hazard evaluation for sprcific seismic regions of the world: Tectonophysics, **194**, 153-169.

Tsapanos. T. M., Lyubushin. A. A, and Pisarenko, V. F., 2001. Application of a Bayesian Approach for estimation of seismic hazard parameters in some regions of the circum-Pacific belt: Pageoph, **158**, (in press).

Wallace, P., 1970. Earthquake recurrence intervals on San Andreas Fault: Bull. Geol. Soc. Am., **81**, 2875-2990.

Yegulalp, T. M. and Kuo, J. T., 1974. Statistical prediction of occurrence of maximum magnitude earthquakes: Bull. Seismol. Soc. Am., **64,** 393-414.