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Magnetic Interpretation of Horizontal Cylinders Using Displacement of the Maximum and Minimum by Upward Continuation

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Abstract: *Three points belonging to maximum / minimum anomalies are obtained for infinitely extended horizontal cylinders. However, only two points will be prominent on the anomalies and the third will generally be far from the origin. In this paper an interpretation technique is suggested using displacement of the maximum and minimum when the upward continuation of the magnetic anomaly is performed. The depth (z), the index parameter (Q) and the amplitude coefficient (C) of the cylinder are calculated by deriving the equations, and the origin is located on the profile. The applicability of the method is tested for a theoretical model and the vertical magnetic anomaly over the Pima copper mine in Arizona is interpreted.*

Keywords: magnetic interpretation; cylinder; upward continuation; shifted of extrema.

INTRODUCTION

Many authors have proposed several methods to find the depth, the index parameter and the amplitude coefficient due to infinitely extended horizontal cylinders, which represent a class of geological structures. Parker Gay (1965) provided a set of master curves, Mohan et al. (1982), Ram Babu and Atchuta Rao (1991) used the Hilbert transform, and Mohan et al. (1990) used the Mellin transform in interpreting magnetic anomalies of the cylinders. Rao et al. (1973), Prakasa Rao et al. (1986) and Abdelrahman (1990) proposed different techniques for horizontal cylinders.

Radhakrishna Murthy (1985) interpreted the magnetic anomaly due to dikes and faults using displacement of the midpoint of the maximum and minimum anomalies if anomalies were continued to a height h. In this case the midpoint shifted a small distance, whereas the maximum and minimum were displaced larger than the midpoint. In this paper we use the displacements of the maximum and minimum for the interpretation of the magnetic anomalies due to infinitely extended horizontal cylinders.

MAGNETIC ANOMALY EXPRESSION

The buried horizontal cylinder considered here extends to infinity along the y-axis. The normal section of the cylinder is in the x-z plane and the origin of the coordinate system is vertically above the center of the cylinder on the surface. The ordinate z denotes the depth to the center of the cylinder, and x is the abscissa of observation point (Fig. 1).

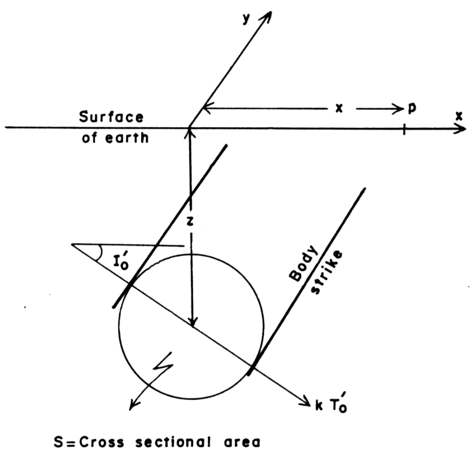

FIG. 1. Geometry of the long horizontal cylinder arbitrarily magnetized by induction in the earth's field .

The general expression for the magnetic anomaly (vertical, horizontal and total) observed at a point P along the x-axis due to an infinitely extended horizontal cylinder is given by (Prakasa Rao et al. (1986).

$$\Delta F(x) = C\frac{(z^2 - x^2).\cos Q + 2zx.\sin Q}{(z^2 + x^2)^2} \qquad (1)$$

where C and Q (their actual values are given in Table 1) are the amplitude coefficient and the index parameter, respectively.

Table 1. Equivalent values of the amplitude coefficient, C and the index parameter, Q in the vertical (ΔV), horizontal (ΔH) and total (ΔT) magnetic anomalies for a long horizontal cylinder**.**

Anomaly	Amplitude Coefficient(C)	Index Parameter(Q)
ΔV	$2kT_0' S$	$I_0'-90^o$
ΔH	$2kT_0'S \sin\alpha$	$I_0'-180^o$
ΔT	$2kT_0'S \sin I_0 / \sin I_0'$	$2I_0'-180^o$

In Table 1, k is the magnetic susceptibility contrast, S is the cross-sectional area and α is the strike of the cylinder measured clockwise from magnetic north. T_0' and I_0' are the values of effective total intensity and inclination, and they are related to real total intensity, T_0 and real inclination, I_0 by

$$T_0' = T_0 \sin I_0 / \sin I_0'$$

and

$$I_0' = \tan^{-1}(\tan I_0 / \sin\alpha)$$

ANALYSIS

Since at the maximum and minimum values of a function the first derivative is equal to zero, differentiating equation (1) with respect to x and setting it equal to zero, the condition for the extremum of ΔF can be obtained as,

$$x^3 - (3.z.\tan Q).x^2 - (3.z^2).x + z^3.\tan Q = 0 \qquad (2)$$

The solutions of equation (2) are :

$$x_{0,1} = z \tan(Q/3) \qquad (3a)$$

$$x_{0,2} = z \tan[(Q-\pi)/3] \qquad (3b)$$

and

$$x_{0,3} = z \tan[(Q+\pi)/3] \qquad (3c)$$

The points $x_{0,1}$ and $x_{0,2}$ are the principal abscissas where the $\Delta F(x)$ takes the maximum and minimum values. The third solution $x_{0,3}$ is generally far from the origin, so we do not take in consideration.

If the anomalies are continued to height h, the position of maximum and minimum anomalies are shifted (Fig**.** 2)**.**

The principal roots on the upward continued profile are,

$$x_{h,1} = (z + h) \tan(Q/3) \qquad (4a)$$

and

$$x_{h,2} = (z + h) \tan[(Q-\pi)/3] \qquad (4b)$$

From equations (3a-b) and (4a-b), one obtains

$$Q = 3 \tan^{-1}[(x_{h,1} - x_{0,1})/h] \qquad (5a)$$

and

$$Q = 3 \tan^{-1}[(x_{h,2} - x_{0,2})/h] + \pi \qquad (5b)$$

Since, by virtue of equation (5a-b), one obtains Q in the range 0^0 to 90^0 only, the actual value of Q may be obtained using the following criteria:

- Major positive anomaly towards positive x-axis: $Q = Q_N$;
- Major positive anomaly towards negative x-axis: $Q = -Q_N$;
- Major negative anomaly towards positive x-axis: $Q = 180+Q_N$;
- Major negative anomaly towards negative x-axis: $Q = 180 - Q_N$;

where Q_N is the Q value obtained from equation (5a-b).

Since the index parameter Q is known, the depth z can be obtained from equations (3-a-b) or (4-a-b) as follows:

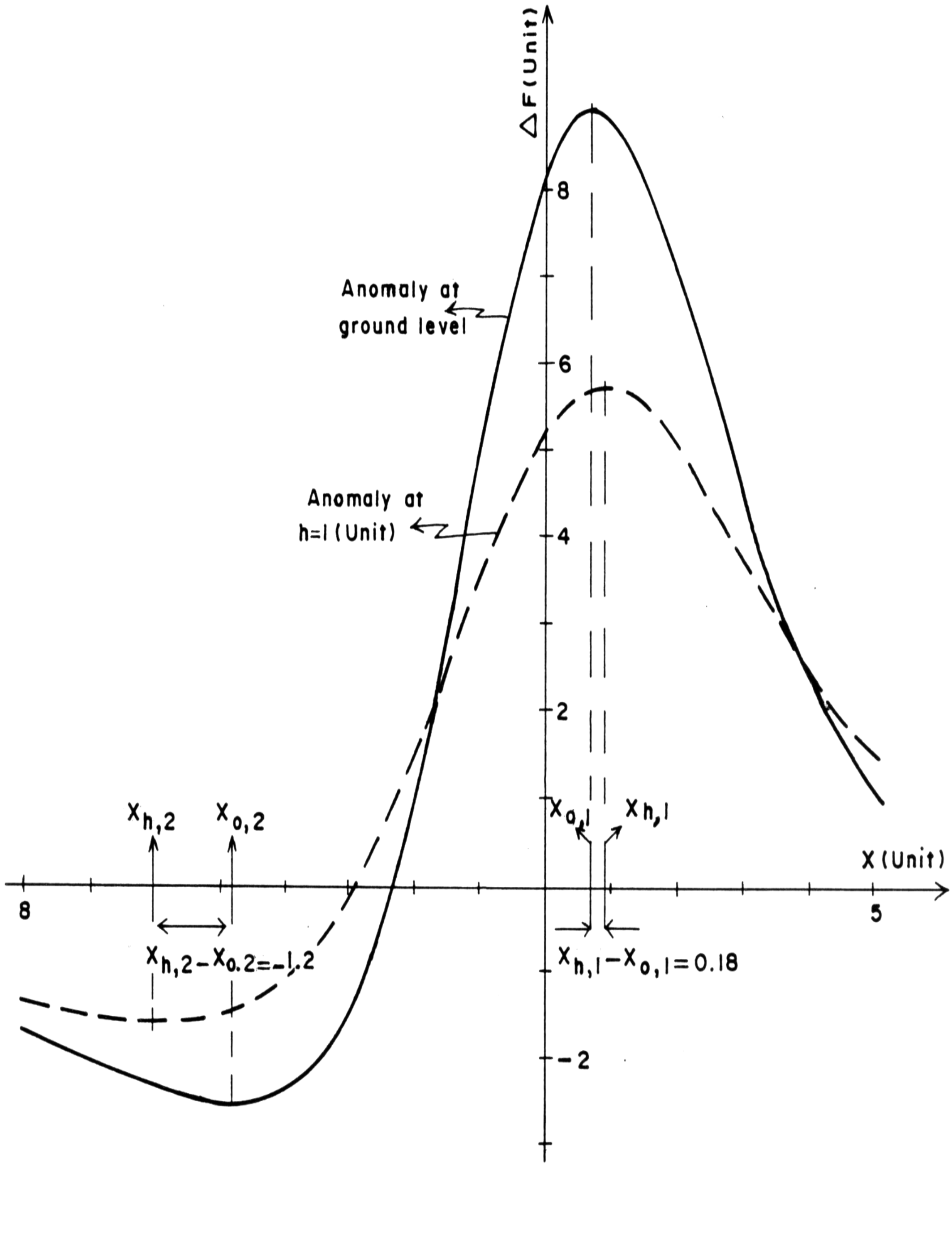

FIG. 2. Magnetic anomaly profile at two levels over a long horizontal cylinder of z = 4 units, Q = 30^0 and C = 150 units for the theoretical example.

$$z = (x_{0,1} - x_{0,2})/\{\tan(Q/3)-\tan[(Q-\pi)/3]\} \quad \text{or} \qquad (6a)$$

$$z = (x_{h,1} - x_{h,2})/\{\tan[(Q-\pi)/3] - \tan[(Q-\pi)/3]\}-h \qquad (6b)$$

The origin is located at a distance -z tan (Q/3) from the abscissa of the maximum anomaly or z tan [(Q-π) /3] from the abscissa of the minimum anomaly of the ground level profile (3a, 3b).

If the observation point is in origin, (x = 0), then equation (1) can be written,

$$\Delta F(0,0) = C \cos Q / z^2 \qquad (7a)$$

for ground level profile, and

$$\Delta F(h,0) = C \cos Q / (z + h)^2 \qquad (7b)$$

for h level profile. From equations (7a) and (7b), one obtains the amplitude coefficient C as

$$C = \frac{\Delta F(0,0) - \Delta F(h,0)}{\cos Q \cdot \left[\frac{1}{z^2} - \frac{1}{(z+h)^2} \right]} \qquad (8)$$

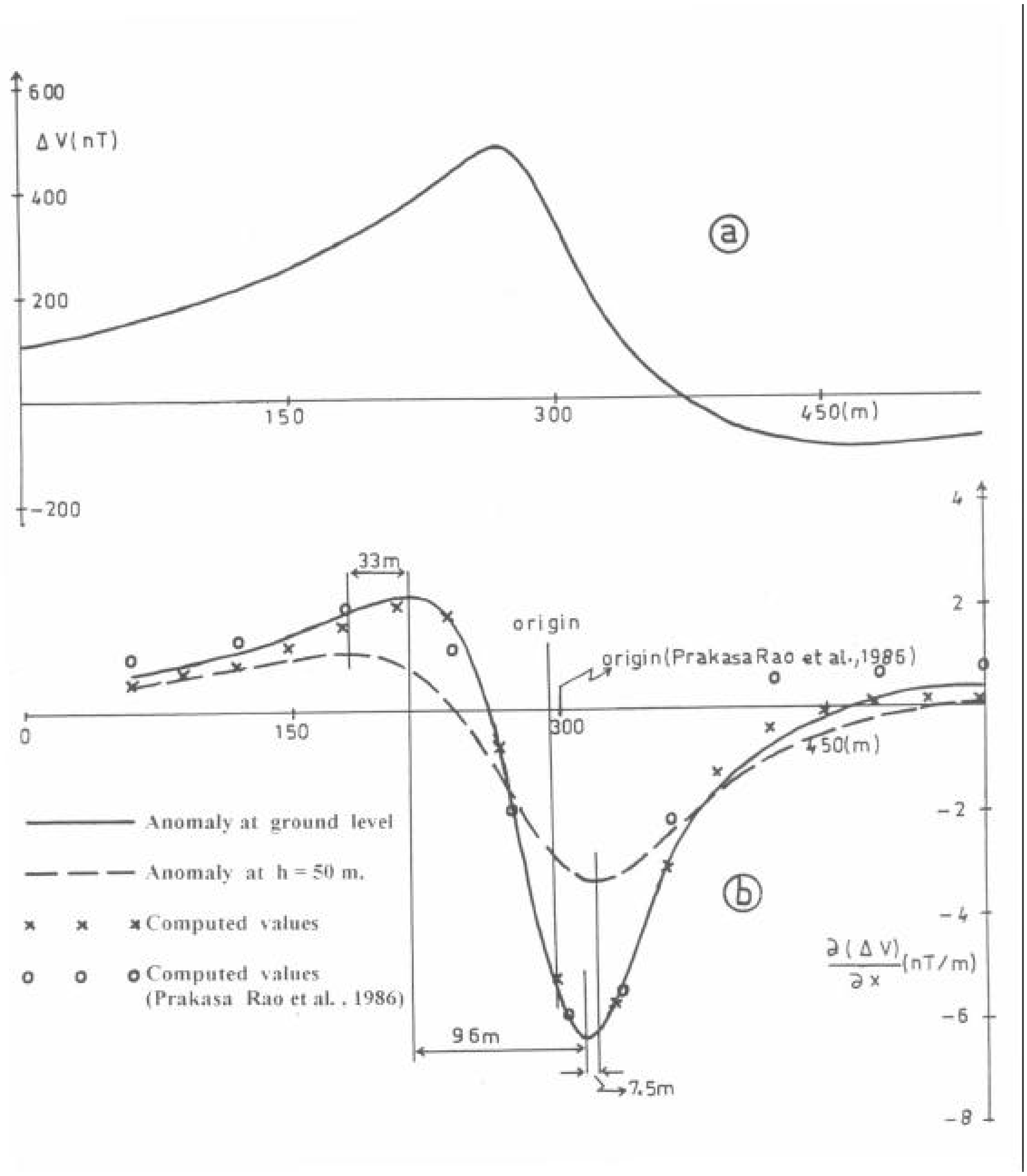

FIG. 3-a. The vertical magnetic anomaly over Pima copper mine in Arizona.
-b. and its horizontal derivative anomaly [After Prakasa Rao et al. (1986)].

THEORETICAL EXAMPLE

The magnetic anomaly due to an infinitely extended horizontal cylinder, whose source parameters are shown in Table 2, is depicted with a solid line (Fig. 2). The analytical upward curve of this anomaly at h=1 level appears as dashed line. The results $x_{h,1}$-$x_{0,1}$ = 0.18 and $x_{h,2}$-$x_{0,2}$ = -1.20 are found by marking the maximum and minimum of those anomalies. From equation (5a) , we get Q = 30.6^o and from equation (5b), we get Q = 29.4^o. Its average value is 30^o. The depth is then calculated from equation (6a) as z = 4.02 units and the location of the origin is marked as -0.709 units from maximum using -z tan(Q/3). By measuring $\Delta F(0,0)$-$\Delta F(h,0)$ = 2.95 units, the amplitude coefficient is found as C = 153.4 units by equation (8). The assumed and evaluated values are shown in Table 2.

Table 2. Theoretical example (in arbitrary units)

Parameters	C	Z	Q
Assumed values	150.0	4.00	30^o
Evaluated values	153.4	4.02	30^o

FIELD EXAMPLE

We consider the magnetic anomaly over the Pima copper mine in Arizona, analyzed by Parker Gay (1963), where the assumed source is a thin dike of infinite depth (Fig. 3a). Prakasa Rao et al. (1986) re-analyzed it. Since the horizontal derivative of the vertical magnetic anomaly due to a thin dike and the vertical magnetic anomaly caused by the horizontal cylinder are identical in shape, the first horizontal derivative of the Pima copper vertical magnetic anomaly (Fig. 3b) has been reinterpreted by our method. When the analytical upward continuation reaches h=30 m level, $x_{h,1}$-$x_{0,1}$ = 7.5 m and $x_{h,2}$-$x_{0,2}$ = -33 m are found. From equation (5a), we get Q=42^o+180^o=222^o and from equation (5b), we get Q=36^o+180^o=216^o. Its average value is 219^o. The depth is then calculated from equation (6a) as 73 m and the location of the origin is marked as -16.9 m from the maximum anomaly. The amplitude coefficient is found as C=39 nT/m by equation (8).

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