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Nomograms for Interpretation of Gravity Anomalies of a Vertical Cylinder

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Abstract: *Nomograms are generated to determine the top and bottom boundaries of a geological structure using the residual gravity anomalies of a vertical cylinder. The nomograms are estimated using the corresponding axis values ($x_{3/4}$ and $x_{1/4}$) of the ¾ g_{max} and ¼ g_{max} , which are determined from the residual gravity anomaly. In addition, a small computer code based on equations representing the nomograms provides simple and fast interpretation. The amplitude coefficient can be easily determined due to the top and bottom depths of the vertical cylinder are known from the interpretation of the residual gravity anomaly. The validity of the method is approved with synthetic data sets and then applied to observed data set.*

Keywords*: Nomograms, gravity interpretation, vertical cylinder.*

INTRODUCTION

The geological structures we are looking for using the geophysical exploration tools, in general, are approximated by simple shaped geometrical models. The cylinder is one of the most popular geometrical models. In these studies, the cylinder usually extends horizontally (e.g. Gay, 1965; Odegard and Berg, 1965; Abdelrahman, 1990; Abdelrahman et al., 1989). However, the target geological structures could also be in the form of a cylinder extending downwards. Nettleton (1942) and Abdelrahman et al. (2001) interpreted a cylinder with a bottom boundary extending to infinity.

Here, a new method is developed to interpret the residual gravity anomaly of a cylinder extending downward to a finite depth by generating nomograms. The method should be applied on the residual gravity anomaly for acquiring reliable results. The nomograms can be represented with polynomial expressions that are useful for interpretation of the anomalies through a small computer program. The validity of the nomograms and the computer program are tested on both noiseless and noisy data sets. In addition, the method is applied to the observed data set, which is acquired along a profile crossing a salt dome in Texas, Harris Country by Abdelrahman et al. (2001).

FORMULATION AND PREPARATION OF THE NOMOGRAMS

According to Nettleton (1942) the gravity at an observation point, *P*, caused by a vertical cylinder given in Fig.1 is expressed with the following relation;

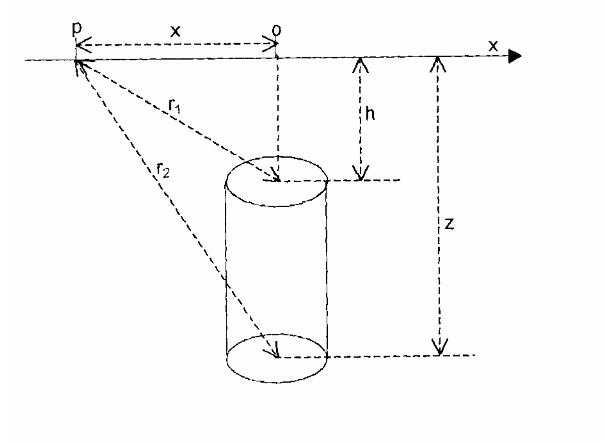

Fig. 1. Vertical cylinder extending to a finite depth.

$$g(x) = A\left[\frac{1}{\left(x^2 + h^2\right)^{0.5}} - \frac{1}{\left(x^2 + z^2\right)^{0.5}}\right] \quad (1)$$

Here, A is the amplitude coefficient, $A=\pi R^2 G\rho$, h is the depth to the upper part of the body, z is the depth to the bottom part of the body, R is the radius of the cylinder, ρ is the density contrast and G is the gravity constant.

The gravity anomaly of the vertical cylinder is illustrated in Fig 2. whose origin is under the g_{max} value. If the horizontal axis values of the ¾ g_{max} and ¼ g_{max} are $x_{3/4}$, $x_{1/4}$, respectively, then the $x_{1/4}/x_{3/4}$ ratio is independent from the amplitude coefficient and depends only on h and z.

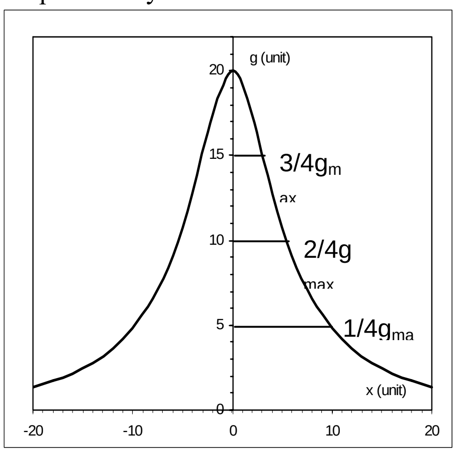

Fig. 2. The gravity anomaly and critical points of the vertical cylinder extending to a finite depth.

We have noticed that for each z/h ratio there is a specific $x_{1/4}/x_{3/4}$ ratio. Using this fact and taking $h=1$ the relation (1) can be rewritten in terms of $x_{3/4}$ as,

$$\frac{3g_{\max}}{4}=\left[\frac{1}{\left(x_{3/4}{}^{2}+1^{2}\right)^{0.5}}-\frac{1}{\left(x_{3/4}{}^{2}+z^{2}\right)^{0.5}}\right]$$

$$\frac{3g_{\max}}{4}=\left[\frac{\left(x^{2}+z^{2}\right)^{0.5}-\left(x^{2}+1^{2}\right)^{0.5}}{\left(x^{2}+z^{2}\right)\left(x^{2}+1^{2}\right)}\right] \qquad (2)$$

(3)

where $x = x_{3/4}$ and rearranging the terms in equation (3) we obtain,

$$0.28125g^2x^4+0.28125g^2x^2 + 0.28125g^2x^2z^2 + 0.28125g^2z^2 - x^2 - 0.5z^2 - 0.5 = -(x^4 + x^2 + x^2z^2 + z^2)^{1/2} \qquad (4)$$

where $g = g_{max}$ and finally, the last form of the equation is obtained as,

$$(b^2g^4)x^8+[2b^2g^4(1+z^2)-2bg^2]x^6+[b^2g^4(4z^2+z^4+1)-3bg^2(1+z^2)]x^4+[2b^2g^4z^2(1+z^2)-bg^2z^2(4+z^2)-bg^2]x^2+[b^2g^4z^4-bg^2z^2(1+z^2)+z^2(0.25z^2-0.5)+0.25] = 0 \qquad (5)$$

where $b = 0.28125$.

By applying the same procedure to the equation (1), the following equation can be easily obtained in terms of $x_{1/4}$,

$$g^4x^8+[2g^2(g^2+g^2z^2-32)]x^6+[g^2(g^2-96)+g^2z^2(g^2z^2+4g^2-96)]x^4+[2g^2z^2(g^2z^2+g^2-16z^2-64)-32g^2]x^2+[g^2z^2(g^2z^2-32z^2-32)+256z^2(z^2-2)+256] = 0 \qquad (6)$$

These equations are solved for different z values using the well-known Matlab program. The $x_{3/4}$ and $x_{1/4}$ values are calculated from these solutions and the results of $x_{1/4}/x_{3/4}$ ratios are given in Table I.

Table I. $x_{1/4}/x_{3/4}$ and $z/x_{1/4}$ ratios versus different z/h ratios

z/h	$x_{1/4}/x_{3/4}$	$z/x_{1/4}$
1.1	2.68337	0.851
1.2	2.68923	0.890
1.4	2.70823	0.964
1.6	2.73360	1.036
1.8	2.76269	1.104
2.0	2.79380	1.171
3.0	2.95394	1.482
4.0	3.09873	1.773
5.0	3.22240	2.053
6.0	3.32711	2.326
7.0	3.41613	2.595
8.0	3.49238	2.861
9.0	3.55817	3.125
10.	3.61546	3.388

Using the corresponding $x_{1/4}/x_{3/4}$ ratio for each z/h ratio, a nomogram is constructed and illustrated in Fig. 3.

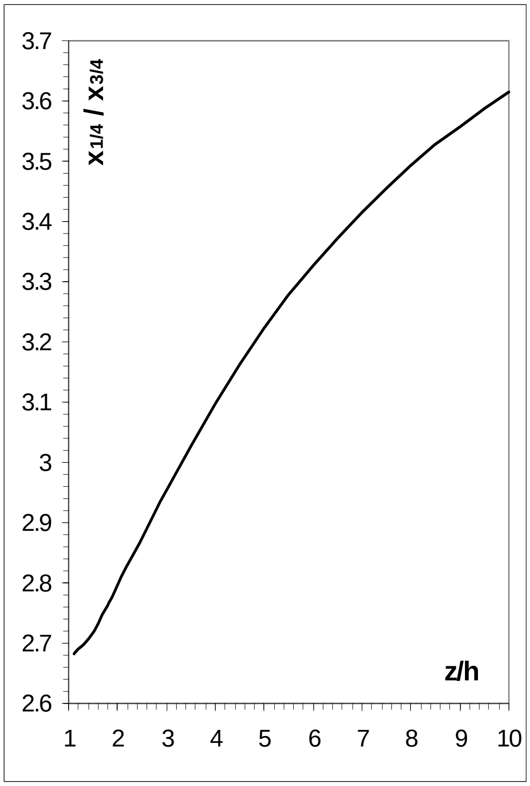

Fig. 3. Nomogram of variation in $x_{1/4}/x_{3/4}$ versus different z/h ratio for a vertical cylinder

Since we know the *z* and *h* values in the *z/h* ratio a nomogram for $z/x_{1/4}$ ratio is also constructed (Fig. 4).

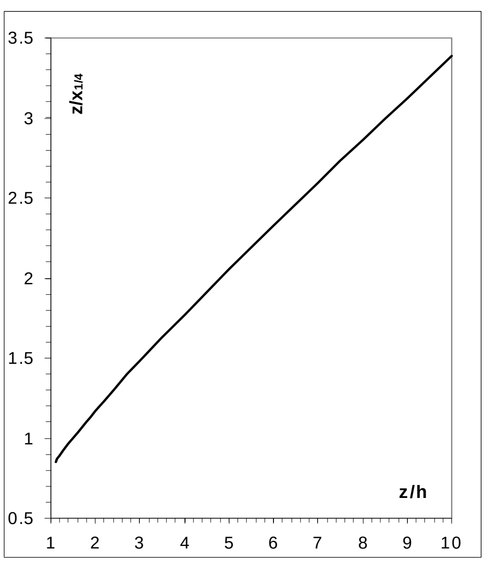

Fig. 4. A Nomogram to determine the bottom depth (z) of a vertical cylinder

Either these nomograms or the polynomial expressions for $x_{1/4}/x_{3/4}$ and $z/x_{1/4}$ can be utilized in the interpretation of the anomalies.
The polynomial expression for the $x_{1/4}/x_{3/4}$ ratio is

$$zh=a+a1.x+a2.x^2+a3.x^3+a4.x^4+a5.x^5+a6.x^6 \quad (7)$$

where,
a= -39.65967 , a1=19.20202 , a2=-0.8754978 , a3=0.6498856 , a4= -0.2774661 ,
a5= -0.109835 , a6=0.03413242 zh=z/h and $x=x_{1/4}/x_{3/4}$
The polynomial expression for the $z/x_{1/4}$ ratio is

$$zx14=b+b1.zh+b2.zh^2 \quad (8)$$

where,
b=0.522275 , b1=0.32412 , b2=0.003753 and $zx14=z/x_{1/4}$

Thus, the interpretation of the anomalies is simplified using a computer program (see the appendix), which is based on the relation's (7) and (8).

APPLICATION

The method is applied in the following way:

1) The maximum value of the vertical cylinder is marked. This value is just above the origin.
2) The $x_{3/4}$ and $x_{1/4}$ locations of the ¾ g_{max} and ¼ g_{max} are determined.
3) The $x_{1/4}/x_{3/4}$ ratio is determined. Using this ratio and Fig. 3, the *z/h* ratio of a vertical cylinder is determined.
4) To obtain z, the vertical axis value in Fig. 4.corresponding to the *z/h* ratio obtained in step 3 is multiplied with the length of $x_{1/4}$. Since *z/h* ratio has been previously determined, then *h* can be easily obtained.
5) Finally the amplitude coefficient (*A*), which remains as an unknown parameter in equation (1), can be easily calculated.

If this procedure is applied to a sphere, it is noticed that the determined $x_{1/4}/x_{3/4}$ ratio is very close to the $z/h \cong 1.1$ ratio of the vertical cylinder.

SYNTHETIC MODELS

The reliability of the method is tested with a theoretical model whose anomaly is shown in Fig.2. Assumed values and evaluated values are given in Table II.

Table II. Theoretical example for a vertical cylinder (in arbitrary units)

	Parameters	h	z
Vertical Cylinder	Assumed values	4.00	20.00
	Evaluated values	3.95	20.16
	Evaluated values with adding random noise (%5)	3.88	17.10

Here, $x_{3/4}=3.0$ and $x_{1/4}=9.7$ are determined from the anomaly which yields the ratio $x_{1/4}/x_{3/4}=3.2$. Using this value and Fig. 3.the ratio of z/h is obtained as 5.1. By using this result and Fig. 4.the z and h values are obtained as given in Table II. The gravity anomaly of the vertical cylinder, with 5 % random noise added, can be seen in Fig.5. Here, $x_{3/4}=3.0$ and $x_{1/4}=9.5$ are determined from the anomaly. The results of the method applied to the noisy data are also given in Table II.

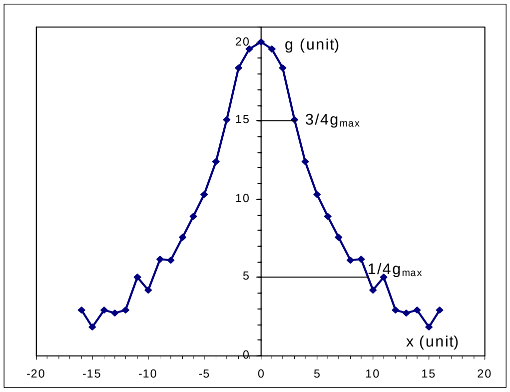

Fig. 5. A gravity anomaly of the vertical. cylinder extending to a finite depth with 5 % noise added

FIELD EXAMPLE

We re-interpret the residual anomaly extracted from the Bouguer gravity map of the Humble salt dome by Abdelrahman et al. (2001) using the method presented in this study.

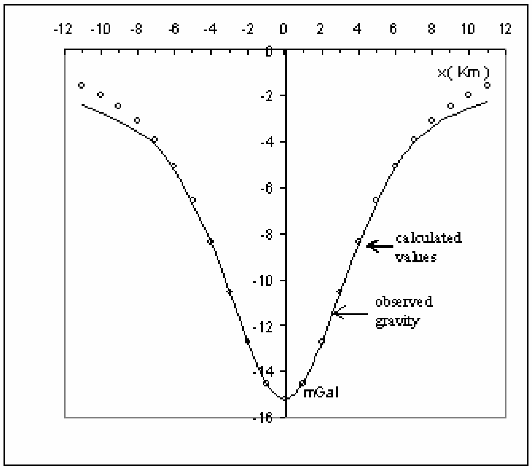

Fig. 6. Residual (Abdulrahman et al. 2001) and calculated gravity anomaly profiles of Humble salt dome, Harris County, Texas.

The results are $x_{3/4}=2.63$ km, $x_{1/4}=7.23$ km which yield the ratio $x_{1/4}/x_{3/4}=2.75$ ($z/h=1.65$). Then, using the Fig. 4.it is estimated that $z=7.59$ km. and $h=4.58$ km. Comparison of the calculated depth results in the present study with the results of the other studies performed in the same field is given in Table III.

Table III. Comparison of the calculated depth results.

Authors	Depth h (km)	Depth z (km)
Nettleton (1976)	4.97	-
Mohan et al. (1986)	4.96	-
Abdelrahman and El-Araby T.M. (1993)	4.92	-
Abdelrahman et al. (2001)	4.96	-
Present study	4.58	7.59

CONCLUSION

In this study, a new methodology is introduced for the interpretation of the residual gravity anomaly of a vertical cylinder extending to a finite depth. By using this new method, both the top and the bottom depths of a vertical cylinder can be determined easily. Using the axes values of the ¾ g_{max} and ¼ g_{max} of the residual gravity anomaly makes interpretation easier and faster than conventional methods. The key point are eliminating the regional contribution and estimating the zero line of the residual anomaly precisely.

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APPENDIX

"A basic computer code for determining the top and the bottom depths of a vertical cylinder"

```
10  INPUT "X3/4=", X34
20  INPUT "X1/4=", X14
30  X= X14/X34
40  A = -39.65967 : A1=19.20202 :  A2= -0.8754978
50  A3 = 0.6498856 : A4= -0.2774661 : A5= -0.109835
60  A6 = 3.413242E-02
70  ZH = A+A1*X+A2*X^2+A3*X^3+A4*X^4+A5*X^5+A6*X^6
80  PRINT "Z/H="; ZH
90  B= 0.522275 : B1= 0.32412 : B2= -0.003753
100  ZX14 = B+B1*ZH+B2*ZH^2
110  Z = ZX14*X14
120  H = Z/ZH
130  PRINT "Z=";Z
140  PRINT "H=";H
150  END
```