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Wavelet denoising of magnetic prospecting data

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ABSTRACT: *A denoising scheme is presented suitable for magnetic exploration data. It is based on an empirical study and combines the shift invariant denoising method with the use of a variable threshold, which is calculated in each cycle of the shift invariant algorithm. The calculation of each threshold is based on the wavelet transformation of random noise with the same standard deviation as in the data. The scheme is particularly useful in "archaeological geophysics" where it is important to produce results that mostly resemble the plane view of the buried objects.*

Keywords: *Wavelet transformation, multiresolution analysis, magnetic data, denoising.*

INTRODUCTION

Data collected in a magnetic exploration are often contaminated with noise and artifacts coming from various sources. The presence of noise in data distorts the characteristics of the signal resulting in poor quality of any subsequent processing. Consequently the first step in any processing of such data is the "cleaning up" of the noise in a way that preserves the signals sharp variations.

There are several ways for noise reduction. The most common and therefore classical denoising methods are the local polynomial regression, the adaptive filtering, the optimum Wiener filtering and the moving average filters. In the last decade the wavelet transform, due to localization property, has become a powerful signal and image processing tool, and it has found applications in many scientific areas. An excellent example has been given by Donoho and Johnstone (1994), who proposed a novel wavelet, based denoising method by shrinking noisy wavelet coefficients via thresholding.

This method is based mainly on two properties of the wavelets: they concentrate the energy of a smooth signal in a few wavelet coefficients, and the transformation of white noise is also white noise. Therefore it is reasonable to assume that small coefficients represent the noise and can be set to zero, while the large ones contain the signal's energy and should be kept.

The work of Donoho and Johnstone (1994) comprises the basis of a class of denoising methods in wavelet domain (Vidacovic, 1999; Donoho, 1995; Coifman and Donoho, 1995; Smith and Dentith, 1999; Krim et al., 1999; Kovac, 1998). In general, the wavelet domain denoising method has the advantage that it preserves the signal's characteristics. The performance of the method is affected by the choice of the wavelet, the thresholding function and the threshold selection rule. There is no unique denoising policy covering all data types. Each data type needs an empirical selection of the proper combination of the above mentioned issues.

The present paper deals with denoising of magnetic data in wavelet domain, and is organized as follows. A brief description of the wavelet transforms is given in the second paragraph. In the third paragraph, the basics of the wavelet denoising method are presented while paragraph summarizes the results of an empirical study of denoising of magnetic data in wavelet domain. The main outcome of the present study is the proposed wavelet denoising scheme. It combines the shift invariant cycle-spinning algorithm proposed by Coifman and Donoho (1995), and a variable threshold calculated in each cycle of the algorithm. Finally, the performance of the proposed scheme is tested, at first on synthetic data and then on real data collected from exploration of archaeological sites. The results show a significant suppression of the noise and a very good recovery of the original signal.

WAVELET TRANSFORMS

A. The continuous wavelet transform

Wavelets are families of functions $\psi_{a,b}(t)$ defined as dilations and translations in time of ψ named mother wavelet. The family of functions $\psi_{a,b}(t)$ has the form

$$\psi_{ab}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), b \in R, a > 0, \qquad (1)$$

where a and b are the scaling and translation parameters. The normalization factor $1/\sqrt{a}$ preserves that $\|\psi_{a,b}\| = 1$. *The continuous (integral) wavelet transform* of a function $f(t) \in L^2(R)$, where $L^2(R)$ denotes the vector space of the square integrable functions, is defined as

$$w(a,b) = \langle f, \psi(a,b) \rangle = \int_{-\infty}^{\infty} f(t)\psi^*_{a,b}(t)dt, \qquad (2)$$

where $\psi^*_{a,b}(t)$ is the complex conjugate of $\psi_{a,b}(t)$.

The inverse wavelet transform is defined as

$$f(t) = c_{\psi}^{-1}\iint_{\infty} w(a,b)\psi_{a,b}(t)\frac{dadb}{a^2}, \qquad (3)$$

where

$$C_{\psi} = \int_{-\infty}^{\infty}\frac{|\psi(\omega)|^2}{\omega}d\omega < \infty, \qquad (4)$$

and $\Psi(\omega)$ is the Fourier transform of $\psi(t)$ (Vidacovic, 1999).

B. Discrete wavelet transform - Multiresolution analysis

According to Mallat (1989) a multiresolution analysis is a family $(V_j)_{j\in Z}$ of closed subspaces of $L^2(R)$ with the following properties:

- $V_j \subset V_{j+1}$
- $g(t) \in V_j \Leftrightarrow g(2t) \in V_{j+1}$
- $g(t) \in V_0 \Leftrightarrow g(t-1) \in V$

- $\bigcup_{j\in z} V_j$ is a dense in $L_2(R)$ and $\bigcap_{j\in z} V_j = \{0\}$.

- There is a function $\varphi \in V_0$, named father wavelet or scaling function, such that the family $\{\varphi(x-k): k \in Z\}$ is an orthogonal basis of V_0. The family

$$\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k), \ \mathrm{j,k} \in \mathrm{Z}$$
(5)

is an orthogonal basis of V_j, and the family

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k), \ \mathrm{j,k} \in \mathrm{Z},$$
(6)

is an orthogonal basis of the subspace W_j, where W_j is the orthogonal complement of V_j to V_{j+1}, defined as

$$V_{j+1} = V_j \oplus W_j.$$
(7)

Consequently the space V_{j+1} can decomposed as

$$V_{j+1} = V_0 \oplus W_{0+} \oplus W_1 \oplus W_2 + .. \oplus W_j .$$
(8)

The sequences $\{V_j\}_{j\in Z}$ and $\{W_j\}_{j\in Z}$ are called approximation and detailed spaces respectively. The projection of a signal on level V_{j+1} is defined as a sum consisting of the projection on the level V_j and the projection on the W_j.

Since $\varphi(t)$ is an element of V_0, we can express the function $\varphi(t)$ as

$$\phi(t) = \sqrt{2}\sum_{-\infty}^{\infty} h[n]\phi(2t-n),$$
(9)

where the coefficients h_n define a discrete low-pass filter.

Also since $\psi(t)$ is an element of W_0, we can write

$$\psi(t) = \sqrt{2}\sum_{-\infty}^{\infty} g[n]\psi(2t-n),$$
(10)

where the coefficients $\mathbf{g_n}$ define a discrete high-pass filter.
The successive computation of the discrete wavelet transform (DWT) of a function *f(t)* is described by the recursive formula

$$\alpha_j(k) = \sum_n h[2k-n]a_{j+1}, \ \mathrm{d_j}(k) = \sum_n g[2k-n]a_{j+1}$$
(11)

The coefficients a_j are called approximation coefficients, and the coefficients d_j detail coefficients. Details regarding the wavelet analysis can be found in the works of Mallat (1998) and Kaiser (1994).

WAVELET DENOISING METHOD

The main wavelet denoising scheme is summarized as follow:

- Wavelet transformation of data.
- Estimation of the noise level and use of it to threshold the wavelet coefficients
- Reconstruction of the estimation of the signal from the shrunk wavelet coefficients

All shrinkage (thresholding) methods nullify the wavelet coefficients with absolute value less than a threshold. Depending on how the coefficients are treated when they are larger than the threshold, we can define different shrinkage methods. The shrinkage method and the selection of the threshold define the quality of the signal recovering.

Thresholding functions. The most common thresholding methods are *soft* and *hard*. The analytical expressions of hard and soft thresholdings are

$$\delta^h(d,\lambda) = \begin{cases} d & |d| > \lambda, \lambda \geq 0 \\ 0 & |d| \leq \lambda \end{cases}, \quad (12)$$

$$\delta^s(d,\lambda) = \begin{cases} (d - sgn(d).\lambda & |d| > \lambda, \lambda \geq 0 \\ 0 & |d| \leq \lambda \end{cases} \quad (13)$$

Threshold selection rules.

The selection of the threshold is very important. A very small threshold does not remove the noise. On the other, a very large threshold creates shrinkage of the signal. For some rules set, the threshold depends on the shrinkage function, while in other alternative rules the calculation of the same threshold for different shrinkage functions is proposed. In general, the selection is based on the standard deviation of the noise or the more robust mean absolute deviation (MAD) proposed by Donoho (1995). The most common rules for threshold selection are:

- The *universal threshold* proposed by Donoho (1992).
- The *Sure* threshold selection rule (Donoho and Johnstone, 1995). This threshold and the soft shrinkage function are the core of the adaptive level dependent *SureShrink* rule.
- The optimal *minimaxi* fixed threshold proposed by Donoho and Johnstone (Vidacovic, 1999). It can be regarded as an improvement of the universal threshold.
- Ridsdill-Smith and Dentith (1999) proposed for the wavelet denoising of aeromagnetic data a threshold selection policy based on the Monte Carlo analysis of the statistical behavior of the wavelet transformation of Gaussian white noise (same standard deviation as in data).

In general there is no a unique denoising policy covering all data cases. Each data category needs empirical selection of the proper combination of the wavelet, the thresholding function, the decomposition levels and of the threshold selection rule. For instance, the rules *Sure* and *minimaxi* are suitable for signals with small details in the region of the noise. For signals with high frequency oscillations, Krim et al. (1999) suggests a best basis selection for the signal's transformation with the use of wavelet packets and cosine transformations.

Translation invariant wavelet denoising. Traditional thresholding of the wavelet coefficients sometimes, in the neighborhood of discontinuities, exhibits Gibbs phenomena due to the lack of translation invariance of the wavelet basis. To eliminate these phenomena, Coifman and Donoho (1995) proposed a wavelet denoising strategy called cycle spinning denoising algorithm. For a number of circular shifts of the signal, the algorithm performs the following actions:

- Denoises the shifted data.
- Unshifts the denoised data.
- Averages the obtained results.

This method produces a reconstruction of the signal exhibiting much weaker Gibb's phenomena.

EMPIRICAL STUDY OF THE WAVELET DENOISING OF MAGNETIC DATA

As we have mentioned in the third paragraph, there is not unique wavelet denoising strategy suitable for all data types. In denoising literature with wavelets and in wavelet signal processing toolboxes one can find wavelet denoising functions whose parameters are the wavelet, the thresholding function, the noise level and the threshold rule and type (stable or level dependent).

Our study of the wavelet denoising of geophysical magnetic data is empirical. We used synthetic data from a model of either one or two adjacent prismatic bodies. The model's signal was corrupted by Gaussian white noise with standard deviation 1 or 2. The signal's length used is of 32 samples which can be regarded as the maximum dyadic length of the geophysical profile. We avoided data expansion by interpolation, because this procedure distorts the noise. Next, we performed denoising using all possible combinations of the parameters in each function. The signal's reconstruction quality was evaluated by the mean standard deviation defined with the aid of the difference of each data point from the estimated one. The conclusions of the empirical study are summarized bellow:

- *Soft Thresholding disorders the signal for all function's used.*
- *The wavelets Coiflet1 and Daubechies2, tend to give comparatively better results. We did not use wavelets with a large number of coefficients, because of the sort profile length.*
- *The shift invariant algorithm combined with hard thresholding and definition of the standard deviation (std) of noise, gives the most reliable results.*
- *In case of magnetic anomaly data, we cannot assume that the standard deviation of the noise is estimated by the standard deviation of the wavelet coefficients of the finest level.*

Based on the above mentioned conclusions we propose the following wavelet denoising procedure:

1. Symmetrical extension of data in order to have dyadic length.
2. Wavelet transformation of data.
3. Threshold selection, and hard thresholding of the coefficients.
4. Reconstruction of the signal.
5. Repetition for a set of cyclic shifts of data and evaluation of the mean value

The selection of the variable threshold has the following steps

1. Generation of white Gaussian noise with same standard deviation as in data.
2. Wavelet transformation of the noise with the Coiflet1 wavelet. Evaluation the mean value m and the standard deviation σ of the noise coefficients in the finest level of the transformation.
3. Select as threshold the value $(m+2.5\sigma)$.

Due to the randomness of the noise, the repetition of the threshold selection procedure in each signal's shift, gives a different threshold in each case. The averaging of the results of all shifts gives a better estimation of the signal.

EXAMPLES

Synthetic example

As an example we present the denoising of the anomaly of two adjacent prismatic bodies whose upper surface is buried at 1m depth. This signal was corrupted with normal white noise at two different levels with standard deviation σ=1 and σ=2.88

respectively. The signal to noise ratio (SNR) is 10.8 and 19.23. We also performed denoising using the Wiener filtering. The quality of the signal's estimation is justified by the signal to noise ratio SNR of the recovered anomaly, and is presented in Table 1.

It is evident from Table 1 that the proposed scheme produces a better estimation of noise level in both cases. On the contrary, the Wiener filtering shows a rather poor performance, even thought the exact signal's model has been used. Figure 1 shows the coincidence of the original signal and its estimation computed by the proposed scheme.

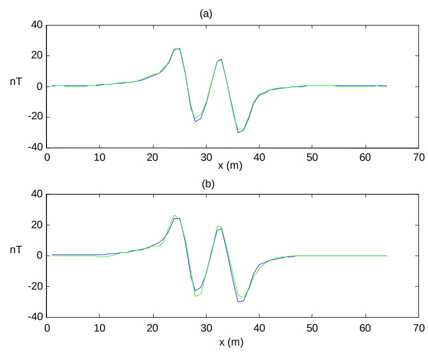

FIGURE 1. Coincidence of the signal recovery to the actual signal. The initial signal and the recovered one are depicted by the blue and green lines respectively. In the upper part (case a), the initial signal was contaminated by white noise having standard deviation 1 while the noise level was increased to standard deviation 2 for the case 2 (lower part of the figure). Evidently, the attempted matching is very good.

Denoising of magnetic total field data from the exploration of the archaeological site of Europos in Northern Greece

Europos was a commercial center on the banks of the river Axios in Northern Greece (Region of Macedonia). The ruins of the ancient urban center and installations are hosted in the subsurface, near a modern village that bears the same name.

The data subjected to denoising are shown in figure (2).

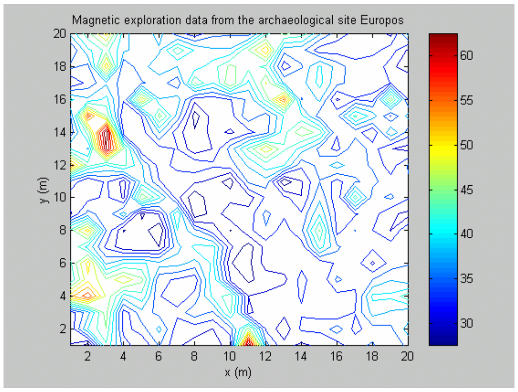

FIGURE 2. Magnetic total field data from the exploration of the archaeological site Europos in N. Greece. Contours are in nT .The particular 20x20 m grid comprises a small part of the geophysical mesh (Tsokas et al, 1994) which has been extracted for the demonstration of the proposed denoising scheme. Evidently the distribution of the total field suffers from high frequency noise. On the other hand a lot of information about the subsurface situation is also present.

They comprise a 20x20 m grid extracted from the mesh established in the site for the acquisition of geophysical data (Tsokas et al., 1994). The collection of readings was performed along profiles spaced 1 m apart each from the other, stepwise at 1m intervals. Part of the ruins of the ancient urban complex are concealed in the subsurface under the particular bit of land. These ruins of foundation walls would had shown up if a gray scale or dot density image had been constructed for these data. Nevertheless, we chose the presentation in the form of contour map because it demonstrates better the effect of denoising of the field. The noise was modeled as white Gaussian with standard deviation of σ=2. The application of the new wavelet denoising scheme resulted in the distribution of the Earth's total magnetic field shown in figure (3).

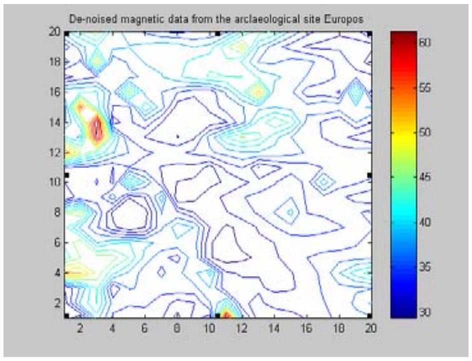

FIGURE 3. Denoised version of the total magnetic field data of the grid shown in figure (2) from the exploration of the archaeological site of Europos (N. Greece). Contours are in nT. The enhancement of the image of the raw data is evident.

For better illustration of the method's performance a single profile was extracted from the grid and plotted by solid line in figure (4).

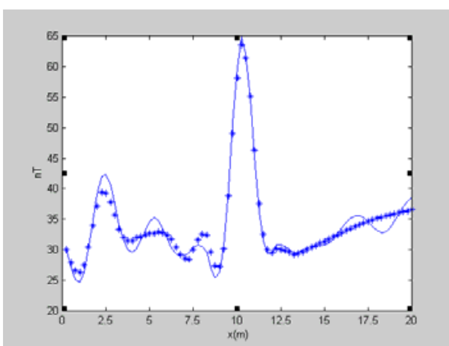

FIGURE 4. The solid line depicts the total magnetic field variation along a single profile of the grid of figure (4). The denoised data are depicted by stars.

The denoised readings along the same profile are depicted by stars in the same figure. It is obvious that the wavelet method gives satisfactory results without affecting the anomalies that carry the information for the subsurface. The noisy details have disappeared resulting in a smooth version of the field.

Denoising real data from the magnetic survey of Makrygialos in Northern Greece (Pieria).

As a second field example we use the data collected in the magnetic survey of the archaeological site of Makrygialos in N. Greece (region of Macedonia). The results of the survey have been reported by Tsokas et al. (1997). The site hosts the remnants of a Neolithic settlement. Again, we used a small fraction of the whole data set consisting in a 40x40 m grid. The data acquisition parameters are the same as in the case of the example of Europos.

The noise was modeled as white with standard deviation 2. The map of the original data set is shown in figure (5).

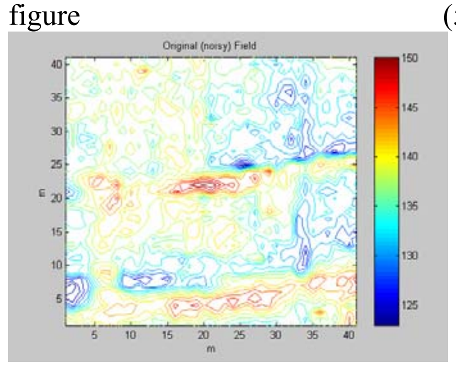

FIGURE.5. Magnetic data from the exploration of the archaeological site of Makrygialos in Northern Greece (Tsokas et al., 1997). Contours are in nT. The grid is part of the broader geophysical mesh. The site conceals the remnants of a Neolithic settlement. The magnetic signatures of two parallel ditches dominate the map. Also, high frequency noise and micro relief induced noise are present.

Contour map presentation was also preferred for the same reasoning as in the case of the previous example.

The effect of two parallel ancient ditches is clearly seen. These data proposed scheme and the results are shown in figures (6).

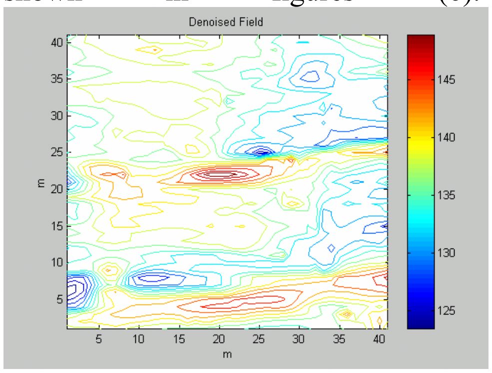

FIGURE 6. Denoised version of the total magnetic field data shown in figure 5. Contours are in nT.

Note that the weak anomalies ranging perpendicular to the anomalies of the ditches have been eliminated. These anomalies are due to small undulations of the surface relief caused by plowing. Further the distribution of the field has been clearly denoised. The data along a particular profile of the grid are shown in figure (7) both in the original form and in the denoised version. The suppression of small noisy artifacts is obvious.

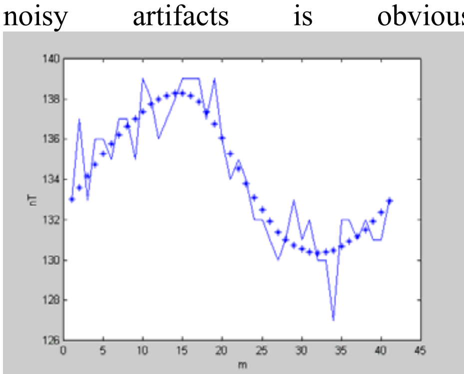

FIGURE 7. Plot of a single profile of the data shown in figures (5) and (6). The solid line and the stars depict the raw and the denoised data respectively.

where denoised with the use of the

DISCUSSION AND CONCLUSIONS

The proposed denoising scheme in wavelet domain tends to give an estimation of the original signal with a significant suppression of the noise. In general, the wavelet denoising method preserves the characteristics of the signal, but it has the disadvantage of requiring relative large dyadic length for the profile. As the profile's length becomes larger, we can use more multiresolution levels for thresholding. The method is also sensitive to gross errors. They have large values and therefore they are not suppressed in the wavelet domain.

There is no unique denoising policy, in wavelet domain. In case of magnetic total field data we applied a cycle-spinning denoising scheme combining hard thresholding and a variable threshold selection procedure based on the wavelet transformation of the noise. It seems that the particular scheme functions satisfactorily.

REFERENCES

Coifman R., Donoho D., 1995: Translation Invariant De-Noising. Yale University and Stanford University: www-stat.stanford.edu/reports/donoho, 2002

Donoho D., 1992: Wavelet shrinkage and W.V.D. A 10-minute tour: www-stat.stanford.edu/reports/donoho, 2002.

Donoho D., 1995: De-noising by soft thresholding. IEEE Trans.Inform.Theory, 41: 613-627.

Donoho D., I. Johnstone, 1995: Adapting to unknown smoothness via wavelet shrinkage, Journal of the American Statistical Association, Vol. 90, N_o 432: 1200-1224.

Kaiser G, 1994: A friendly Guide to Wavelets. Birkhauser.

Kovac A., 1998: Wavelet Thresholding for Unequally Spaced Data. PhD. Thesis, University of Bristol 1998.

Krim H., Tucker D., Mallat S. and Donoho D., 1999: On de-noising and best Signal Representation. IEE Transactions on Information Theory, Vol.45, No 7, November 1999.

Mallat S., 1989: A theory for multiresolution signal decomposition. The wavelet representation. IEEE Trans. on

Donoho D. and Johnstone I., 1994: Ideal spatial adaptation via wavelet shrinkage. Biometrica Vol. 81: 425-455
Pat. Anal. Mach. Intel., 11(7): 674-693.

Mallat S., 1998: A wavelet tour of signal processing. Academic Press, second addition.

Ridsdill-Smith T.A and Dentith M.C., 1999: The wavelet transform in aeromagnetic processing. Geophysics Vol.64: 1003-1013.

Tsokas G. N, Giannopoulos A., Tsourlos P., Vargemezis G., Tealby J., Sarris A., Papazachos C.B., Savopoulou T., 1994: A large scale geophysical survey in the Archaelogical site of Europos (Northern Greece). Journal of Applied Geophysics 32: 85-98.

Tsokas G. N., Sarris A., Pappa M., Bessios M., Papazaxos C. B., Tsourlos P. and Giannopoulos A., 1997: A large scale magnetic survey in Makrygialos (Pieria), Greece. Archaeological Prospection 4(3): 123-128.

Vidakovic B., 1999. Statistical Modeling by Wavelets. John Wiley & Sons.

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